Experimental Study of the Statistical Characteristics of the Scintillation of a Terrestrial Light Source

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Abstract

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A. S. Gurvich, V. I. Tatarskii, and L. R. Tsvang

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The phenomena of scintillation and twinkling of stars and other sources of radiation, located both outside the limits of the Earth' s atmosphere and within it, are at present attracting considerable attention in connection with problems of observational astronomy and certain problems of radio meteorology. A fairly detailed theory of these phenomena has been developed ([1–7]), based on the concepts of the theory of locally isotropic turbulence ([8–11]).

In the case where the fluctuation of the refractive index n of the medium obeys the "2/3 law" ([10,11,7]),

$$\overline{[n(\mathbf{r}+\vec{\rho})-n(\mathbf{r})]^2} = C_n^2 \rho^{2/3} \tag{1}$$

(where C_n^2 is a certain constant quantity depending on $\operatorname{grad} \overline{n}$ and on the characteristics of the turbulence), and when the conditions $\lambda \ll l_0$, $\lambda^3 L \ll l_0^4$, $l_0 \ll \sqrt{\lambda L} \ll L_0$, $C_n^2 L l_0^{-1/3} \ll 1$ are satisfied (where l_0 is the inner scale, L_0 the outer scale of turbulence, λ the wavelength, and L the distance traversed by the wave in the turbulent medium*), the following conclusions of the theory are valid ([5–7]):

1. Fluctuations of the light intensity are distributed according to a logarithmically normal law.
2. The variance of the intensity I of the light wave is described by the formula

$$\sigma^2 = \overline{[\ln I - \overline{\ln I}]^2} = 10.5\, C_n^2 \lambda^{-7/6} L^{11/6}, \tag{2}$$

from which it follows that $\sigma^2 \sim L^{11/6}$.

3. The correlation function B_I of fluctuations of the logarithm of the light intensity in the plane perpendicular to the ray depends on $\rho/\sqrt{\lambda L}$:

$$B_I = B_I\left(\frac{\rho}{\sqrt{\lambda L}}\right), \tag{3}$$

where ρ is the distance between observation points, and the correlation scale is of order $\sqrt{\lambda L}$.**

4. The frequency spectrum of fluctuations is a function of the quantity $f\sqrt{\lambda L}/v_n$, where f is the frequency and v_n is the component, perpendicular to the "ray," of the mean

* In the atmosphere $l_0 \sim 1$ cm, $C_n \sim 10^{-8} \div 10^{-9}$ cm$^{-1/3}$, $L_0 \sim 10 \div 100$ m,

therefore for light the indicated conditions are practically always fulfilled if $L \geq 200$–500 m.

** The assertion contained in ([12]) that the radius of correlation of fluctuations of the wave amplitude is always of the order of the size of the inhomogeneities is valid only in the exceptional case when $l_0 \approx L_0$ (this is precisely the case considered in ([12])). In the case, however, when $l_0 \ll \sqrt{\lambda L} \ll L_0$, the radius of correlation of fluctuations of the amplitude has order $\sqrt{\lambda L}$, independently of the choice of the correlation function of the refractive-index fluctuations.

wind velocity:

$$fW(f) = F\left(\frac{f\sqrt{\lambda L}}{v_n}\right). \tag{4}$$

Here $W(f)$ is the spectral density of the intensity fluctuations, normalized by the condition $\int_0^\infty W(f)df = 1$, and is the Fourier transform of the autocorrelation coefficient of the fluctuations of I.

All these dependences were tested experimentally. The experiments were carried out in 1956–1957 near the settlement of Tsimlyansky over a very flat section of steppe*. From a modulated-light source at a height of about 2 m above the ground, a slightly divergent beam of light was propagated in the direction of the light receiver. The distance L between them took the values 250, 500, 1000, and 2000 m. Through two diaphragms of diameter 2 mm, the distance ρ between which could be varied from 5 to 500 mm, the light fell on FEU-19 photomultipliers. The voltages from the outputs of the photomultipliers were amplified by resonant amplifiers with a passband of about 2000 Hz and detected.

The voltages V_1 and V_2 at the detector outputs, proportional to the light intensity on the diaphragms, were subjected to statistical analysis by means of specially developed apparatus, which made it possible to measure directly: a) $\overline{V_1}$; b) $\overline{[V_1 - \overline{V}_1]^2}$; c) the frequency spectrum of the fluctuations; d) the probability distribution of the fluctuations $V_1' = V_1 - \overline{V}_1$; e) $\overline{(V_1' - V_2')^2} = D^2$. The latter quantity made it possible to calculate the correlation coefficient of the fluctuations $R = 1 - D^2/2\,\overline{(V_1 - \overline{V}_1)^2}$.

Specially modified electrodynamic voltmeters were used for squaring. For frequency analysis a special frequency analyzer was developed with a range from

0.05 to 1160 Hz. The probability distributions were studied with a 25-channel analyzer. The apparatus automatically ensured the equality $\overline{V}_1 = \overline{V}_2$.

Simultaneously with the scintillation measurements, measurements were made of the mean temperature and wind velocity at heights of 0.5, 1, 2, 4, 8, and 12 m, and of the wind direction. On the basis of these measurements it is possible to determine the characteristics of the turbulent regime of the atmosphere ([13,14]), which were used in analyzing the measurement results.

Results of measurements

1. Distribution function of the light-intensity fluctuations. To test the hypothesis of normality of the distribution, the empirical distribution functions were plotted on a scale in which this law is expressed by a straight line (one such graph is shown in Fig. 1). About 100 empirical distribution functions were processed. All of them are in satisfactory agreement with the hypothesis of a logarithmically normal distribution law for I. Using this law, the quantity $\sigma^2 = \overline{[\ln I - \overline{\ln I}]^2}$, which enters the theory (see (2)), can be expressed through experimentally observed quantities

$$\sigma^2 = \ln\left[1 + \frac{\overline{(I-\overline{I})^2}}{\overline{I}^2}\right] = \ln\left[1 + \frac{\overline{(V_1-\overline{V}_1)^2}}{\overline{V}_1^2}\right]. \quad (5)$$

Formula (5) was used in the subsequent processing of the experimental data.

2. Dependence of the magnitude of scintillation on distance. In the experiments described, no simultaneous

* L. V. Terentyeva took part in the measurements.

measurements of σ^2 for different values of L. Therefore, before comparing the values of σ^2 obtained both for different L and under different meteorological conditions, it is necessary to eliminate the latter factor. The simplest way of reducing the observational data to identical meteorological conditions is to average all values of σ^2 obtained for a given L and under different meteorological conditions. The values obtained in this way are shown in Fig. 2,

(Figure: Figure 1 and Figure 2)

Fig. 1. Integral probability distribution $P(I < n)$ of the light intensity. The scale along the abscissa is logarithmic; along the ordinate is plotted the function inverse to the probability integral.

Fig. 2. Dependence of σ^2 on distance. The numbers on the graph indicate the number of measurements for each distance.

from which it is seen that the dependence of σ^2 on the distance L is in satisfactory agreement with the theoretical $\sigma^2 \sim L^{11/6}$. B. A. Suchkov [15], who studied fluctuations in the propagation of sound in the atmosphere, arrived at analogous results.

3. Correlation function of fluctuations of light intensity. In measuring the correlation coefficient R, the distance ρ between the diaphragms was always set so that the quantity $\rho/\sqrt{\lambda L}$ took the values 1/4; 1/2; 1; 2; 4 and 8. The measured values of R show a comparatively large scatter. However, the values of R averaged at different fixed L agree well with one another. In Fig. 3 the values of R are plotted for different ρ and L as a function of $\rho/\sqrt{\lambda L}$. For large values of $\rho/\sqrt{\lambda L}$, $R < 0$, which is in agreement with the theory. Thus, the results obtained confirm the similarity law $R = R(\rho/\sqrt{\lambda L})$.

Table 1

	$L = 1000$ m	$L = 2000$ m				
	v_n, m/sec	v_n, m/sec				
	1.5	2.2	3.5	1.6	2.6	3.5
f_m, Hz	20	26	46	18	26	40
$f_m\sqrt{\lambda L}/v_n$	0.31	0.26	0.30	0.35	0.31	0.36

4. Frequency spectra of fluctuations of light intensity. About 80 frequency spectra obtained at distances $L = 1000$ m and $L = 2000$ m were processed. The spectral densities corresponding to each of these distances were divided into three groups, depending on the wind-velocity component v_n: 1 m/sec $< v_n <$ 2 m/sec; 2 m/sec $< v_n <$ 3 m/sec; 3 m/sec $< v_n <$ 4 m/sec, and geometrically averaged within each group. In Fig. 4 a group of curves $fW(f)$ obtained in this way is presented. A shift of the frequency spectrum to the right with increasing v_n is clearly seen. According to formula (4), the frequency f_m at which $fW(f)$ assumes its maximum value is determined by the relation $f_m = \text{const} \cdot v_n/\sqrt{\lambda L}$. Table 1 gives the values of v_n, f_m, and $f_m\sqrt{\lambda L}/v_n$. As is seen from the table,

the quantity $f_m\sqrt{\lambda L}/v_n$ is indeed almost constant, which is in good agreement with the similarity law (4). For a more detailed verification of this similarity law, the spectra were plotted on graphs in dimensionless coordinates

(Figure: Fig. 3. Correlation function $R(\rho/\sqrt{\lambda L})$. The vertical line is the 5% confidence interval. a—2000 m, b—1000 m, v—500 m, g—average for all distances)

Fig. 3. Correlation (function) $R(\rho/\sqrt{\lambda L})$. The vertical line is the 5% confidence interval. a—2000 m, b—1000 m, v—500 m; g—average for all distances

(Figure: Fig. 4. Frequency spectra of fluctuations in light intensity. 1—$v_n = 1.5$ m/sec; 2—2.2 m/sec; 3—3.5 m/sec; 4—1.6 m/sec; 5—2.6 m/sec; 6—3.5 m/sec)

Fig. 4. Frequency spectra of fluctuations in light intensity. 1—$v_n = 1.5$ m/sec; 2—2.2 m/sec; 3—3.5 m/sec; 4—1.6 m/sec; 5—2.6 m/sec; 6—3.5 m/sec

$$\ln \frac{f}{f_m}, \qquad \frac{f}{f_m} W\left(\frac{f}{f_m}\right).$$

The results of such processing gave additional confirmation of the similarity law (4).

To summarize, it may be noted that the experimental data we obtained are in satisfactory agreement with the principal conclusions of the theory formulated above.

Institute of Atmospheric Physics
Academy of Sciences of the USSR

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CITED LITERATURE

1. V. A. Krasil’ nikov, A. M. Obukhov, *Akust. zhurn.*, **2**, issue 2 (1956).
2. A. A. Krasil’ nikov, *DAN*, **65**, No. 3 (1949).
3. V. A. Krasil’ nikov, *Izv. AN SSSR, ser. geogr. i geofiz.*, **13**, No. 1 (1949).
4. A. M. Obukhov, *Izv. AN SSSR, ser. geofiz.*, No. 2 (1953).
5. V. I. Tatarskii, *DAN*, **107**, No. 2 (1956).
6. V. I. Tatarskii, *DAN*, **120**, No. 2 (1958).
7. V. I. Tatarskii, Dissertation, Acoustics Institute, Academy of Sciences of the USSR, 1956.
8. A. N. Kolmogorov, *DAN*, **30**, No. 4 (1941).
9. A. M. Obukhov, *DAN*, **32**, No. 1 (1941).
10. A. M. Obukhov, *Izv. AN SSSR, ser. geogr. i geofiz.*, No. 1 (1949).
11. A. M. Yaglom, *DAN*, **69**, No. 6 (1949).
12. L. A. Chernov, *Akust. zhurn.*, **1**, issue 1 (1955).
13. A. S. Monin, A. M. Obukhov, *Tr. Geofiz. inst. AN SSSR*, No. 24 (151) (1954).

14. V. I. Tatarskii, *Izv. AN SSSR, ser. geofiz.*, No. 6 (1956).

15. B. A. Suchkov, *Akust. zhurn.*, **4**, issue 1 (1958).

Note: Figure translations are in progress. See original paper for figures.