AERODYNAMICS

1958

SovietRxiv

Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

AERODYNAMICS

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A FINITE-SPAN WING IN A COMPRESSIBLE FLOW WITH A SYMMETRIC PROFILE

(Presented by Academician L. I. Sedov, 3 January 1958)

1. Let us consider the motion of a thin wing of finite span with an angle of attack equal to zero. We shall assume that the principal motion of the wing is a translational rectilinear motion with, in general, variable velocity, and that it takes place within an unbounded volume of compressible fluid at rest at infinity. We shall assume that the planform of the wing is arbitrary, and that the wing surface is symmetric with respect to the plane of motion of the wing ([1,2]).

Small additional oscillatory motions, in which the wing surface is deformed, may be superposed on the principal motion of the wing. In this case the oscillatory motions take place in such a way that at every instant of time the wing surface preserves its symmetry with respect to the plane of motion of the wing.

Let us take a right-handed system of rectilinear coordinate axes $Oxyz$, rigidly connected with the space in which the motion of the wing takes place. We direct the axis Ox along the direction of motion of the wing, and arrange the plane xOy so that the coordinates z of the points of the wing surface are small (Fig. 1).

Fig. 1

The law of the principal motion of the wing is given in the form

$$x = F(t), \tag{1}$$

where F is an arbitrary continuous function of time; for definiteness, let the coordinate x be the coordinate of some fixed point O_1 on the leading edge of the wing.

The normal component of velocity on the upper and lower sides of the wing surface, respectively, is prescribed by the law:

$$\varphi_n = A, \qquad \varphi_n = -A, \tag{2}$$

where A is a function of time and of the points on the wing surface, defined as

$$A = A_0 + A_1 \qquad (A_0 = -F'(t)\beta). \tag{3}$$

The functions β and A_1 are prescribed at each point on the wing surface. These functions are small and are arbitrary integrable functions of their arguments. The first term in the expression for A corresponds to the principal motion of the wing, and the second to the additional unsteady motions.

We shall regard the velocity potential φ of the perturbed motion of the fluid and its derivatives as small quantities of the first order and neglect small quantities of the second order. We consider the problem in a linearized formulation.

The velocity potential, as is known, satisfies the wave equation

$$\varphi_{xx} + \varphi_{yy} + \varphi_{zz} - \frac{1}{a^2}\varphi_{tt} = 0, \tag{4}$$

where a is the speed of sound in the undisturbed medium.

We transfer the boundary conditions on the wing surface parallel to the Oz axis onto the projection Σ of the wing on the fixed plane xOy. Thus, on the basis of the prescribed law (2), we obtain the flow conditions

$$\varphi_z = A(x, y, t), \tag{5}$$

$$\varphi_z = -A(x, y, t), \tag{6}$$

which must be satisfied respectively on the upper and lower sides of Σ.

Everywhere in the plane xOy outside the region Σ the condition is satisfied

$$\varphi_z(x, y, 0, t) = 0. \tag{7}$$

It is sufficient to solve the problem for the upper half-space. The velocity potential in the lower half-space will be found as
$\varphi(x, y, -z, t) = \varphi(x, y, z, t)$, since φ is an even function for rectilinear motion of the wing. Thus, the boundary-value problem consists in the following: to find a function $\varphi(x, y, z, t)$ that satisfies equation (4), the boundary conditions (5), (7), and whose derivatives vanish at infinity: $\lim \varphi_x = \lim \varphi_y = \lim \varphi_z = 0$ as $r \to \infty$, where $r = \sqrt{x^2 + y^2 + z^2}$.

Fig. 2

Fig. 2

Figure 2: Fig. 2

Fig. 3

Figure 3: Fig. 3

2. To solve the boundary-value problem we apply the method set forth in works ([3],[4]).

Let us turn to the space of the variables x, y, t. Consider the region V (Figs. 2 and 3). The region V is bounded by the surface Σ^*, which is the geometric locus of curves representing the laws of motion of the points of the wing contour. Let the curve AO_1BD, forming the wing contour (Fig. 1), be given by the equation $y_1 = \Psi(x_1)$ in the moving coordinate system $x_1O_1y_1$, where $x_1 = x - F(t)$, $y_1 = y$. Then the surface Σ^* is determined by the equation $y = \Psi[x - F(t)] = \psi(x, t)$. In the region V, the derivative φ_z is prescribed according to condition (5). Outside the region V, the derivative φ_z is equal to zero according to condition (7).

Fig. 3

The formula that is a solution of equation (4) and establishes the dependence between the velocity potential φ at an arbitrary point of the space (xyz) of motion of the wing and the derivative φ_z, normal to the plane xOy, for any instant of time t, was obtained earlier ([3,4]) and has the form:

$$\varphi(x, y, z, t) = -\frac{a}{2\pi} \iint\limits_{S(x,y,z,t)} \frac{\varphi_z \left\{\xi, \eta, 0, t - \frac{1}{a}\sqrt{(x-\xi)^2 + (y-\eta)^2 + z^2}\right\}}{\sqrt{(1+a^2)(x-\xi)^2 + (1+a^2)(y-\eta)^2 + a^2 z^2}}\, dS, \tag{8}$$

where the integration extends over the surface S—a branch of the hyperboloid

$$a^2(t-\tau)^2 - (x-\xi)^2 - (y-\eta)^2 - z^2 = 0, \tag{9}$$

extending to infinity in the direction of decreasing values of time.

If the derivative φ_z is determined from the boundary conditions (5) and (7), then formula (8) will give the required solution. By formula (8) one can compute the velocity potential for any set of variables x, y, z, t, if in it one sets the derivative $\varphi_z(\xi, \eta, \tau) = A(\xi, \eta, \tau)$ and extends the integration over that part S^* of the surface S which belongs to the region V. On the remaining part $S - S^*$ this derivative is equal to zero. Figure 2 shows the mutual position of the surfaces Σ^* and S for supersonic velocity of motion of the wing; Fig. 3, for subsonic velocity (l^* is the line of intersection of the surfaces S and Σ^*).

Thus, formula (8) gives an effective solution of the problem in the case of symmetric flow past the wing, when the wing moves according to an arbitrary law, both at supersonic and at subsonic velocity.

The problem of flow past a finite-span wing with a symmetric profile and zero angle of attack in a steady supersonic stream was solved in papers ([1,2]).

3. Let the wing move with constant velocity u. In this case the law of motion of the wing has the form $x = F(t) = ut$. The surface Σ^* is a cylindrical surface with generators inclined to the axis Ot at an angle whose tangent is u. In formula (8) we pass from the surface integral to a double integral with a plane domain of integration in the moving plane $x_1O_1y_1$, moving with the velocity u of the wing (Fig. 1). If $u < a$, the velocity of motion of the wing is subsonic, then we obtain the formula

$$-2\pi\varphi(x_1, y_1, z, t) = \iint \frac{\varphi_z\left\{\xi_1, \eta_1, 0, t + \frac{u(x_1-\xi_1)}{u^2-a^2} + \frac{a}{u^2-a^2}\sqrt{(x_1-\xi_1)^2 - k^2(y_1-\eta_1)^2 - k^2z^2}\right\}}{\sqrt{(x_1-\xi_1)^2 - k^2(y_1-\eta_1)^2 - k^2z^2}}\, d\xi_1 d\eta_1, \tag{10}$$

where the variables of integration vary within the limits $-\infty \leqslant \xi_1 \leqslant +\infty$, $-\infty \leqslant \eta_1 \leqslant +\infty$, and $k = \sqrt{u^2/a^2 - 1}$.

If $u > a$, the velocity of motion of the wing is supersonic, then we obtain the known formula (([5]), Chap. I, formula (3.1)):

$$-2\pi\varphi(x_1, y_1, z, t) =$$

$$= \iint \frac{\varphi_z\left\{\xi_1, \eta_1, 0, t + \frac{u(x_1-\xi_1)}{u^2-a^2} + \frac{a}{u^2-a^2}\sqrt{(x_1-\xi_1)^2 - k^2(y_1-\eta_1)^2 - k^2z^2}\right\}}{\sqrt{(x_1-\xi_1)^2 - k^2(y_1-\eta_1)^2 - k^2z^2}}\, d\xi_1 d\eta_1 +$$

$$+ \text{ an analogous term with a minus sign before the radical.} \tag{11}$$

The variables of integration in (11) vary within the limits

$$x_1 - kz \leqslant \xi_1 \leqslant +\infty,$$

$$y_1 - \frac{1}{k}\sqrt{(x_1-\xi_1)^2 - k^2z^2} \leqslant \eta_1 \leqslant y_1 + \frac{1}{k}\sqrt{(x_1-\xi_1)^2 - k^2z^2}. \tag{12}$$

4. In constructing the solution of the problem in a moving coordinate system, in the case of symmetric flow past the wing considered in points 1 and 2, when the wing moves with subsonic velocity, in formula (10) the integration extends over the entire region Σ—the projection of the wing onto the

plane $x_1 O_1 y_1$ (or xOy). If the wing moves with supersonic velocity, then in formula (11) the integration extends only over that part of the region Σ which lies inside the region defined by the inequalities (12).

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Received
1 XII 1957

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Note: Figure translations are in progress. See original paper for figures.