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MECHANICS

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1958

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Abstract

Full Text

MECHANICS

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PROPORTIONAL NAVIGATION AS A PROBLEM OF CYBERNETICS

(Presented by Academician V. I. Smirnov, 27 III 1958)

Proportional navigation was first proposed as a method of homing in 1945^(1,2). The previously known methods of homing—those with zero lead angle⁽³⁾, with constant lead angle⁽⁴⁾, and parallel approach—have a number of substantial shortcomings. Therefore, in recent times the method of proportional navigation has attracted attention^(6–9). However, in these works the treatment is limited to a kinetic study of the relative motion of two points; in most cases numerical-integration methods are used, and results are obtained that do not coincide with reality. In the present paper the dynamics of homing under proportional navigation is investigated, as is the choice of an automatic motion-control system that ensures stable guidance of the object to the target.

Fig. 1. Basic law of proportional navigation and kinematic parameters of motion

The idea of proportional navigation consists in counteracting the tendency toward rotation of the line of sight, or target line, connecting object B with target A , and, consequently, in bringing the trajectory of the object closer to the rectilinear path of parallel approach. The law of proportional navigation is

$$\dot{\psi} = b\dot{\eta}, \quad (1)$$

where ψ is the angle of rotation of the velocity vector of the object; η is the angle of rotation of the target line; b is the navigation constant (see Fig. 1).

Let us compose the differential equations of motion of the object in the horizontal plane, assuming the target speed v_s to be constant in magnitude and direction, the object speed v to be constant in magnitude, and the navigation constant equal to 2, which permits obtaining a solution in closed form (Fig. 1):

$$\begin{aligned}
 mv\dot{\psi} &= (T + C_L v^2)\alpha, & J_z \ddot{\varphi} &= -k_1(v)\beta - k_2(v)\dot{\varphi} + k_3(v)\alpha, \\
 \varphi &= \eta - 90^\circ + \alpha - \gamma, & \psi + \gamma &= \eta, & \dot{\psi} &= 2\dot{\eta}, \\
 \dot{a} &= v_s \cos \eta - v \cos \gamma, & a\dot{\eta} &= v \sin \gamma - v_s \sin \eta.
 \end{aligned} \tag{2}$$

The notation $m, T, C_L, \alpha, J_z, k_1(v), k_2(v), k_3(v), a, \beta$ is the same as in (5); the angle γ is the lead angle between the target line and the velocity vector of the object. Integrating the system of the last four equations (2), we find the trajectory of the object

$$a = a_0 \left[\frac{p \sin \gamma + \sin(\gamma - \varepsilon_0)}{p \sin \gamma_0 + \sin(\gamma_0 - \varepsilon_0)} \right]^{\frac{\mu^2 - 1}{p^2 + 2p \cos \varepsilon_0 + 1}} e^{\frac{2p(\gamma_0 - \gamma) \sin \varepsilon_0}{p^2 + 2p \cos \varepsilon_0 + 1}} \tag{3}$$

and the angular velocity of the line of sight

$$\dot{\eta} = -\dot{\gamma} 0.5\dot{\psi} = \frac{v_s}{a_0} [p \sin \gamma_0 - \sin(\varepsilon_0 - \gamma_0)] \left[\frac{a}{a_0} \right]^{\frac{2(1+p \cos \varepsilon_0)}{p^2 - 1}} e^{\frac{2p(\gamma - \gamma_0) \sin \varepsilon_0}{p^2 - 1}}, \tag{4}$$

where a_0 is the initial value of the distance between the object and the target; γ_0 is the initial value of the lead angle; $\varepsilon_0 = \gamma_0 + \eta$; $p = v/v_s$. Equations (3) and (4) were first obtained by Shpitser (2).

Next, using the first and third equations of system (2), we find the angle of attack α and the angle of turn of the object φ . Substituting into the second equation of system (2) the value of α and the derivatives of the angle φ , we find the law of variation of the rudder deflection angle that ensures motion of the object along the ideal trajectory of proportional navigation:

$$\begin{aligned}
 \beta &= A_1 e^{3B_5(\gamma - \gamma_0)} [p \sin \gamma + \sin(\gamma - \varepsilon_0)]^{3B_4} + \\
 &+ A_2 e^{3B_5(\gamma - \gamma_0)} [p \sin \gamma + \sin(\gamma - \varepsilon_0)]^{4B_4 + 1} [p \cos \gamma + \cos(\gamma - \varepsilon_0)] + \\
 &+ A_3 e^{3B_5(\gamma - \gamma_0)} [p \sin \gamma + \sin(\gamma - \varepsilon_0)]^{3B_4 - 2} [p \cos \gamma + \cos(\gamma - \varepsilon_0)]^2 + \\
 &+ A_4 e^{2B_5(\gamma - \gamma_0)} [p \sin \gamma + \sin(\gamma - \varepsilon_0)]^{2B_4} + \\
 &+ A_5 e^{2B_5(\gamma - \gamma_0)} [p \sin \gamma + \sin(\gamma - \varepsilon_0)]^{2B_4 - 1} [p \cos \gamma + \cos(\gamma - \varepsilon_0)] + \\
 &+ A_6 e^{B_5(\gamma - \gamma_0)} [p \sin \gamma + \sin(\gamma - \varepsilon_0)]^{B_4},
 \end{aligned} \tag{5}$$

where

Fig. 2. Stability boundaries near the target according to the angular velocity of rotation of the tangent to the object trajectory (II) and according to the rudder (I)

Figure 2: Fig. 2. Stability boundaries near the target according to the angular velocity of rotation of the tangent to the object trajectory (II) and according to the rudder (I)

$$B_4 = \frac{2(1 + p \cos \varepsilon_0)}{p^2 + 2p \cos \varepsilon_0 + 1}; \quad B_5 = \frac{2p \sin \varepsilon_0}{p^2 + 2p \cos \varepsilon_0 + 1}; \quad (6)$$

$A_1, A_2, A_3, A_4, A_5, A_6$ are constant coefficients, easily expressible in terms of the initial quantities.

In earlier works ^(6,7), where proportional navigation was considered in the kinematic aspect of the relative motion of two points, the possibility of interception of the target by the object was determined by the magnitude of the angular velocity of rotation of the object's velocity vector, which, for a velocity constant in magnitude, is equivalent to a judgment by the magnitude of the normal acceleration. Such a conclusion was based on the incorrect assumption ⁽⁶⁾ of proportionality of the rudder deflection angle to the angular velocity of rotation of the velocity vector. On the other hand, Locke ⁽⁶⁾, contradicting this conclusion of his own, and after him Adler ⁽⁹⁾, assert that for a real object regarded as a rigid body with a control rudder, a miss is inevitable, which is also incorrect.

Fig. 2. Stability boundaries near the target according to the angular velocity of rotation of the tangent to the object trajectory (II) and according to the rudder (I)

The obtained equations (4) and (5) make it possible to resolve this apparent contradiction. From equation (3) it follows that interception of the target occurs for $p > 1$ and at a lead angle γ_k determined by the equality

$$\operatorname{tg} \gamma_k = \frac{\sin \varepsilon_0}{p + \cos \varepsilon_0}. \quad (7)$$

The angular velocity of rotation of the velocity vector (4) then tends to zero if $1 + p \cos \varepsilon_0 > 0$, and increases without bound in the opposite case. The rudder deflection angle tends to zero if the velocity ratio p lies within the limits

$$\frac{\cos \varepsilon_0 - \sqrt{\cos^2 \varepsilon_0 + 8}}{2} < p < \frac{\cos \varepsilon_0 + \sqrt{\cos^2 \varepsilon_0 + 8}}{2}. \quad (8)$$

If p lies outside these limits, the angle of rudder deflection near the target increases without bound, and interception is impossible. These limiting conditions are shown in Fig. 2. The shaded region includes the values of p for which $\dot{\psi}$

Fig. 3. Dependence of the final angle of rotation of the target line on the initial angle ε_0

Figure 3: Fig. 3. Dependence of the final angle of rotation of the target line on the initial angle ε_0

tends to zero near the target, while the angle of rudder deflection increases without bound,

Fig. 3. Dependence of the final angle of rotation of the target line on the initial angle ε_0

which shows that it is impossible to judge the probability of hitting the target from the normal acceleration. At the same time, this diagram shows that for initial angles ε_0 close to zero, proportional navigation doubles the range of ratios of the speeds of the target and the object that ensure an exact hit on the target. The speed of the target must not be less than 0.5 of the speed of the object. The conditions for stable guidance near the target in $\dot{\psi}$ and in β coincide only for one value, $\varepsilon_0 = \pi$.

In Fig. 3, on the basis of equation 7), the dependence of η_k —the final angle of rotation of the line of sight—on ε_0 is presented for various values of p .

For values $p > 1$, the final angle of rotation of the target line at the moment of interception differs little from the initial value of the angle ε_0 . Consequently, in homing that begins in the forward hemisphere, on head-on courses with small lead angles, the target line almost does not turn during the process of homing. A hit occurs, in contrast to previously known methods of homing, also when approaching the target from the forward hemisphere.

If the speed of the object is less than the speed of the target, $p < 1$, which is the case for an object starting toward the target ⁽¹⁰⁾, a miss is inevitable. The problem of reducing the miss is solved in the following way. The distance between the object and the target will become minimal at the lead angle

$$\operatorname{tg} \gamma_1 = \frac{p - \cos \varepsilon_0}{\sin \varepsilon_0} \tag{9}$$

and will be equal, in relative quantities, to

$$\frac{a_{\min}}{a_0} = \left\{ \frac{p^2 - 1}{[p \sin \gamma_0 + \sin(\gamma_0 - \varepsilon_0)] \sqrt{p^2 - 2p \cos \varepsilon_0 + 1}} \right\}^{\frac{p^2 - 1}{p^2 + 2p \cos \varepsilon_0 + 1}} e^{\frac{2p(\gamma_0 - \gamma_1) \sin \varepsilon_0}{p^2 + 2p \cos \varepsilon_0 + 1}}. \tag{10}$$

The miss decreases as γ_0 and η_0 approach the values corresponding to parallel approach, and also as $p \rightarrow 1$. Thus, for example,

Fig. 4. Kinematic parameters of real motion: B —position of the center of inertia of the object in ideal motion; B_1 —position at the same instant of time in real motion

Figure 4: Fig. 4. Kinematic parameters of real motion: B —position of the center of inertia of the object in ideal motion; B_1 —position at the same instant of time in real motion

for $\gamma_0 = 2^\circ$ and $\eta_0 = 178^\circ$, which is close to motion on reciprocal courses, with an increase of p from $1/6$ to $1/2$, i.e., by a factor of 3, the relative miss decreases by more than 200 times—from 0.009126 to 0.00004285. In homing on reciprocal courses with zero lead angle, in order to reduce the miss it is necessary to have $p = 5 \div 10$, which in practice is rarely feasible.

Passing (see Fig. 4) to the real motion of the object (5), in which the homing head continuously tracks the target and the misalignment parameter δ is measured (the letter p denotes quantities in real motion),

$$\dot{\psi}_p = (2 + \delta)\dot{\eta}_p, \quad (11)$$

we shall compose differential equations in variations. To do this, in system (2) we replace the last two equations by the equivalent system

$$\dot{x} = -v \sin \psi, \quad \dot{y} = v \cos \psi, \quad y_s = y_{s0} + v_s t, \quad (12)$$

where y_s, y_{s0}, v_s are the coordinate, initial coordinate, and velocity of the target.

Fig. 4. Kinematic parameters of real motion: B —position of the center of inertia of the object in ideal motion; B_1 —position at the same instant of time in real motion

Considering system (2) as the equations of ideal motion, we obtain analogous equations of perturbed or real motion if we adopt the steering-control law

$$\beta_p = S_{10}\Delta\psi + S_{11}\Delta\dot{\psi} + S_{13}\Delta\ddot{\psi} + \dots \quad (13)$$

Subtracting system (2) from the system of equations of real motion, we find the equations in variations:

$$\begin{aligned} mv\Delta\dot{\psi} &= (T + C_L v^2)\Delta\alpha, & J_z\Delta\ddot{\varphi} &= -k_2(v)\Delta\dot{\varphi} + k_3(v)\Delta\alpha - k_1(v)[\beta_p - \beta], \\ \Delta\varphi &= \Delta\psi + \Delta\alpha, & \Delta\dot{\psi} &= (2 + \delta)\Delta\dot{\eta} + \dot{\delta}\eta, & \Delta\dot{x} &= -v \cos \psi \Delta\psi, \\ \Delta\dot{y} &= -v \sin \psi \Delta\psi, & \Delta\eta &= \operatorname{arctg} \frac{\Delta y \sin \eta + \Delta x \cos \eta}{a + \Delta x \sin \eta - \Delta y \cos \eta}. \end{aligned} \quad (14)$$

The last four equations (14), with the aid of a computing-and-solving device, make it possible continuously to compute the variation $\Delta\psi$, according to which the rudder is shifted (13). The quantities a , η , ψ , v are known from the solution of the ideal-motion problem; δ is measured continuously. Initial conditions:

$$\Delta\psi_0 = \Delta\eta_0 = \Delta\dot{\eta}_0 = \Delta x_0 = \Delta y_0 = 0, \quad \Delta\dot{\psi}_0 = \delta_0\dot{\eta}_0.$$

The first three equations (14) reduce to one nonhomogeneous differential equation with constant coefficients, analogously to (5), which makes it possible to determine the gain coefficients S_{10} , S_{11} , S_{12} , ensuring stable homing of the object to the target.

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Received
20 III 1958

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