THEORY OF ELASTICITY

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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DIFFRACTION OF A PLANE ELASTIC WAVE BY AN ANGLE

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An elastic medium fills a plane with a reentrant angle of opening $\alpha < \pi$ (Fig. 1). At the instant $t = 0$, at the vertex of the angle O there is an elementary plane longitudinal wave

$$\varphi = S^0\left(t - \theta_0 x + \sqrt{\frac{1}{a^2} - \theta_0^2}\, y\right) \tag{1}$$

$(S^0(\xi) = 1$ for $\xi > 0$; $\quad S^0(\xi) = 0$ for $\xi < 0$; $\quad a$ is the velocity of propagation of the longitudinal wave, θ_0 is a

and it causes a diffraction disturbance which, by the instant t, fills the sector $OABCDO$ of radius at and opening $2\pi - \alpha$, and is described by the longitudinal and transverse potentials φ and ψ, which are homogeneous functions of zero degree of the coordinates x, y, z.

Fig. 1

The problem is solved under the condition that on the sides of the angle there are no tangential stresses and no components of the elastic-displacement vector normal to the boundary, i.e.

$$\begin{aligned} \tau_{xy} &= 0, \quad v = 0 \quad \text{for } y = 0; \\ \tau_{x'y'} &= 0, \quad v' = 0 \quad \text{for } y' = 0. \end{aligned} \tag{2}$$

Mechanically, these conditions correspond to adhesion without friction.

It is not difficult to verify that, under the indicated boundary conditions, a plane longitudinal elastic wave, incident on the boundary, does not produce a reflected plane transverse wave. Therefore, after the collision of the incident wave with the edge, outside the above-mentioned sector only a longitudinal disturbance will occur. Hence it is natural to assume that the diffraction disturbance inside the sector $OABCDO$ will be only longitudinal as well, i.e. to take $\psi \equiv 0$. It is easy to see that, under this condition, on each side of the angle one of the boundary conditions is a consequence of the other.

Indeed, if $\psi \equiv 0$, then

$$\tau_{xy} = 2\mu \frac{\partial^2 \varphi}{\partial x \partial y}, \quad v = \frac{\partial \varphi}{\partial y}, \tag{3}$$

or

$$\tau_{xy} = 2\mu \frac{\partial v}{\partial x}. \tag{4}$$

If $v\big|_{y=0} = 0$, then also $\tau_{xy}\big|_{y=0} = 0$, as was required. The same holds on the side of the angle where $y' = 0$.

Thus the problem has been reduced to finding, inside the sector $OABCDO$, the longitudinal potential φ, satisfying the wave equation

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = \frac{1}{a^2} \frac{\partial^2 \varphi}{\partial t^2} \tag{5}$$

under the condition that on the sides of the angle the normal derivative of φ is equal to zero, while on the circular arcs of the sector $OABCDO$ φ assumes known constant values.

The solution of this problem in elementary functions was constructed in paper [1]. If β is the angle between the axis Ox and the direction of propagation of the plane wave, then, for the position of the incident plane wave indicated in Fig. 1 $(\beta > \alpha,\ \beta = \arccos a\theta_0)$, we obtain

$$\varphi(x, y, t) = \operatorname{Re} \frac{1}{\pi i} \ln \frac{(e^{-\gamma_1 i} - z)(e^{-\gamma_2 i} - z)}{(e^{\gamma_1 i} - z)(e^{\gamma_2 i} - z)} + \frac{\gamma_1 + \gamma_2}{\pi}, \tag{6}$$

where

$$z = \left(e^{-\alpha i} \zeta\right)^{\frac{\pi}{2\pi - \alpha}}, \qquad \zeta = \left(\frac{at}{r} - \sqrt{\frac{a^2 t^2}{r^2} - 1} \right) e^{\theta i}; \tag{7}$$

r, θ are the polar coordinates of points inside the sector $OABCDO$,

$$\gamma_1 = \pi\frac{\beta - \alpha}{2\pi - \alpha}, \qquad \gamma_2 = \pi - \frac{\pi\beta}{2\pi - \alpha}.$$

The displacements and stresses will now be found from the formulas

$$u = \frac{\partial\varphi}{\partial x}, \qquad v = \frac{\partial\varphi}{\partial y},$$

$$\sigma_x = 2\mu\frac{\partial^2\varphi}{\partial x^2} + \lambda\Delta\varphi, \qquad \sigma_y = 2\mu\frac{\partial^2\varphi}{\partial y^2} + \lambda\Delta\varphi, \qquad \tau_{xy} = 2\mu\frac{\partial^2\varphi}{\partial x\,\partial y} \tag{8}$$

and can be calculated at any point of the sector, for any instant of time, with any desired degree of accuracy.

The solution (8) constructed corresponds to an elementary incident discontinuous wave (1), equal to unity behind the front and zero in front of the front, and, together with the latter, determines outside the angle α, in the generalized sense of S. L. Sobolev, a solution of the equations of motion. It enables us, by superposition, to construct under the same boundary conditions the solution of the more general diffraction problem, when a longitudinal plane wave φ of the form

$$\varphi = f\left(t - \theta_0 x + \sqrt{\frac{1}{a^2} - \theta_0^2}\, y\right), \tag{9}$$

where the function f is differentiable and $f(0) = 0$, is incident on the sides of the angle α.

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REFERENCES

[1] S. L. Sobolev, *Transactions of the Seismological Institute, Academy of Sciences of the USSR*, No. 41 (1934).

Note: Figure translations are in progress. See original paper for figures.