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Abstract

Full Text

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FORMS OF DEPENDENCES POSSESSING ADDITIONAL POSSIBILITIES FOR TRANSFORMING NOMOGRAMS WITH AN ORIENTED TRANSPARENT OVERLAY

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In papers ([1],[2]) we found certain forms of dependences whose nomograms with an oriented transparent overlay possess additional possibilities for transformation. We now give other forms possessing analogous properties. It is assumed that the transparent overlay carries only scales, silent scales, or fixed points.

1. Form

$$f_{12} + g_{12}g_{34} + f_{34} = 0 \tag{1}$$

($f_{12}, g_{12}, g_{34}, f_{34}$ are abbreviated notations for the functions $f_{12}(\alpha_1, \alpha_2)$, $g_{12}(\alpha_1, \alpha_2)$, $g_{34}(\alpha_3, \alpha_4)$, $f_{34}(\alpha_3, \alpha_4)$) is obtained as the result of eliminating the auxiliary variable β from the system of equations

$$f_{12} + f_{34} = -\beta, \qquad \lg g_{12} + \lg g_{34} = \lg \beta,$$

which can be represented by a nomogram with an oriented transparent overlay. Owing to the presence on the transparent overlay of two noncoincident fixed points, it proves possible to separate the families of lines in either of the two binary fields (α_1, α_2) or (α_3, α_4) of the fixed plane. Into the equations of the nomogram elements one may introduce additionally 9 parameters. For this purpose write equation (1) in the form

$$\begin{vmatrix} 0 & f_{12} & 1 \\ 1 & g_{12} & 1 \\ \dfrac{g_{34}}{1+g_{34}} & \dfrac{-f_{34}}{1+g_{34}} & 1 \end{vmatrix} = 0, \tag{2}$$

multiply equation (2) by the numerical determinant

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \neq 0, \tag{3}$$

and then expand the resulting determinant with respect to the elements of the third row. As a result one obtains an equation having the form (1), but containing 9 parameters, of which 8 parameters are essential. Important special cases of form (1) are the dependences:

$$\begin{vmatrix} f_1 & g_1 & 1 \\ f_2 & g_2 & 1 \\ f_{34} & g_{34} & 1 \end{vmatrix} = 0, \qquad f_1 = \frac{f_2 + f_{34}}{g_2 + g_{34}},$$

$$f_1 f_2 f_{34} + (f_1 + f_2) g_{34} + h_{34} = 0, \qquad f_1 f_{34} + f_2 f_{34} + h_{34} = 0,$$

representable also by nomograms of aligned points. A new valuable property of nomograms with an oriented transparency for these dependencies is the possibility of separating families of lines in binary fields and broader possibilities for transformation.

2. The form

$$\begin{vmatrix} f_{12} & g_{12} & 1 \\ f_{34} & g_{34} & 1 \\ f_{56} & g_{56} & 1 \end{vmatrix} = 0, \tag{4}$$

representable by a nomogram of aligned points, can be obtained by eliminating the auxiliary variables β, γ, and δ from the system of equations

$$f_{12} - \beta = f_{34} - \gamma = f_{56} - \delta, \quad \lg g_{12} - \lg \beta = \lg g_{34} - \lg \gamma = \lg g_{56} - \lg \delta,$$

which admits the construction of a nomogram with an oriented transparency. Multiplying equation (4) by the numerical determinant (3), we again obtain an equation of type (4). In this way one can introduce 9 additional parameters into the equation of the elements of the nomogram. The advantage of a nomogram with an oriented transparency for dependence (4) consists in greater possibilities for transformation.

3. The system of equations

$$f_{12} - f_7 = f_{34} - f_8, \quad g_{12} - g_7 = g_{34} - g_8 = g_{56} \tag{5}$$

admits the introduction into the equation of the field (α_5, α_6) of one arbitrary function, since equations (5) can be written in the form

$$f_{12} - f_7 = f_{34} - f_8 = T_{56} - \beta, \quad g_{12} - g_7 = g_{34} - g_8 = g_{56} - 0,$$

where T_{56} is an arbitrary function, β an auxiliary variable.

Eliminating the variables α_7 and α_8 from equations (5), we obtain the form

$$f_{12} + \Phi(g_{56} - g_{12}) = f_{34} + F(g_{56} - g_{34}).$$

Important special cases:

$$f_{12} + \Phi\left(\frac{\psi_{56}}{\psi_{12}}\right) = f_{34} + F\left(\frac{\psi_{56}}{\psi_{34}}\right), \quad \psi_{56} = \frac{f_{12} + f_{34}}{\psi_{12} + \psi_{34}}.$$

The last dependence, which is a generalization of a form of the fifth nomographic order with two binary fields

$$\psi_5 = \frac{f_{12} + f_{34}}{\psi_{12} + \psi_{34}},$$

has additional possibilities for transforming the nomogram depicting it, since it can be written in the form

$$\frac{a_{12}\psi_{56} + a_{13}}{a_{22}\psi_{56} + a_{23}} = \frac{(a_{11} + a_{12}f_{12} + a_{13}\psi_{12}) + (-a_{11} + a_{12}f_{34} + a_{13}\psi_{34})}{(a_{21} + a_{22}f_{12} + a_{23}\psi_{12}) + (-a_{21} + a_{22}f_{34} + a_{23}\psi_{34})},$$

where $a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}$ are parameters.

Eliminating the variables α_2 and α_8 from equations (5) and changing the indices, we obtain the form

$$f_2 + f_{34} = F(\alpha_1, g_2 + g_{56}) + \Phi(g_{34} - g_{56}).$$

Important special cases:

$$f_2 + f_{34} = F(\alpha_1, \psi_2\psi_{56}) + \Phi\left(\frac{\psi_{34}}{\psi_{56}}\right), \quad \psi_{56} = \frac{f_1 + \psi_{34}}{f_2 + f_{34}}.$$

Eliminating the variables α_2 and α_4 from equations (5) and changing the indices, we obtain the form

$$F(\alpha_1, g_2 + g_{56}) - f_2 = \Phi(\alpha_3, g_4 + g_{56}) - f_4.$$

Important special cases:

$$F(\alpha_1, \psi_2\psi_{56}) - f_2 = \Phi(\alpha_3, \psi_4\psi_{56}) - f_4, \qquad \psi_{56} = \frac{f_2 - f_4}{\psi_2 f_1 - \psi_4 f_3}.$$

4. The system of equations

$$f_{12} - f_7 = f_{56}, \qquad g_{12} - g_7 = g_{34} \tag{6}$$

permits the introduction into the equations of the fields (α_3, α_4) and (α_5, α_6) of one arbitrary function each, since it can be written in the form

$$f_{12} - f_7 = T_{34} - \beta = f_{56} - 0, \qquad g_{12} - g_7 = g_{34} - 0 = T_{56} - \gamma,$$

where T_{34} and T_{56} are arbitrary functions, and β and γ are auxiliary variables.

Eliminating the variable α_7 from equations (6), we obtain the form

$$f_{56} = f_{12} + F(g_{12} - g_{34}).$$

Important special cases:

$$f_{56} = f_{12} + F\left(\frac{\psi_{12}}{\psi_{34}}\right), \qquad f_{12} + \psi_{12}\psi_{34} + f_{56} = 0.$$

The last dependence is a generalization of the Cauchy form with binary field $f_{12} + \psi_{12}\psi_3 + f_4 = 0$.

Eliminating the variable α_2 from equations (6) and changing the indices, we obtain the form

$$f_1 = F(f_2 + f_{56},\, g_2 + g_{34}).$$

Important special cases:

$$f_1 = F(\varphi_2\varphi_{56},\, \psi_2\psi_{34}), \qquad f_1 = F(f_2 + f_{56},\, \psi_2\psi_{34}),$$

$$f_1 = \frac{f_2 + f_{56}}{g_2 + g_{34}}, \qquad f_1 = \varphi_2\varphi_{56} + \psi_2\psi_{34}.$$

5. The form $f_{56} = f_{12} + f_{34}$ permits the introduction into the equations of the fields (α_1, α_2), (α_3, α_4), (α_5, α_6) of one arbitrary function each, since it can be written in the form

$$(f_{12} + T_{12}) - \beta = T_{34} - \gamma = f_{56} - 0, \qquad T_{12} - \beta = f_{34} - 0 = T_{56} - \delta, \tag{7}$$

where T_{12}, T_{34}, T_{56} are arbitrary functions; β, γ, δ are auxiliary variables. Equations (7) open up a new way of constructing nomograms with a triangular transparent overlay ([3]).

If the use of a transparent overlay is allowed, then we obtain analogous forms containing 9 variables. Thus, for example, the form

$$g_{12} - F(\alpha_7,\, f_{12} - f_{56} + f_9) = g_{34} - \Phi(\alpha_8,\, f_{34} - f_{56} + f_9) \tag{8}$$

and its special cases

$$g_{12} - F\left(\alpha_7, \frac{\varphi_{12}\varphi_9}{\varphi_{56}}\right) = g_{34} - \Phi\left(\alpha_8, \frac{\varphi_{34}\varphi_9}{\varphi_{56}}\right), \qquad \frac{\varphi_{56}}{\varphi_9} = \frac{\varphi_7\varphi_{12} - \varphi_8\varphi_{34}}{g_{12} - g_{34}}$$

permit the introduction of one arbitrary function into the equations of the field (α_5, α_6). The form

$$\alpha_7 = F(f_{12} - f_{56} + f_9,\, g_{12} - g_{34} + g_8) \tag{9}$$

and its special cases

$$\alpha_7 = F\left(\frac{\varphi_{12}\varphi_9}{\varphi_{56}},\, \frac{\psi_{12}\psi_8}{\psi_{34}}\right), \qquad \alpha_7 = F\left(f_{12} - f_{56} + f_9,\, \frac{\psi_{12}\psi_8}{\psi_{34}}\right),$$

$$f_7 = \frac{f_{12} - f_{56} + f_9}{g_{12} - g_{34} + g_8}, \qquad f_7 = \frac{\varphi_{12}\varphi_9}{\varphi_{56}} + \frac{\psi_{12}\psi_8}{\psi_{34}},$$

$$f_7 + f_{56} - f_9 = f_{12} + \psi_{12}\frac{\psi_8}{\psi_{34}}$$

admit the introduction of two arbitrary functions (one in the field equation (α_3, α_4) and one in the field equation (α_5, α_6)). And, finally, the form

$$f_{12} + f_{34} + f_{56} = f_7 + f_8 + f_9 \tag{10}$$

admits the introduction of three arbitrary functions.

Forms (8), (9), and (10) are generalizations of the forms of dependencies with six variables considered in §§ 3, 4, and 5.

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Note: Figure translations are in progress. See original paper for figures.