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STATE OF NUCLEAR
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Abstract

Full Text

PHYSICS

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ON THE POSSIBILITY OF THE APPEARANCE OF A SUPERFLUID STATE OF NUCLEAR MATTER WHEN p - p AND n - n INTERACTIONS ARE TAKEN INTO ACCOUNT

(Presented by Academician N. N. Bogolyubov on 16 VII 1958)

The structure of nuclear matter is in many respects similar to the electronic structure of a metal. It is therefore of interest to apply the mathematical methods successfully developed in the theory of superconductivity ⁽¹⁾ to the study of the properties of nuclear matter.

A superfluid state, i.e., a state lower in energy than the normal state (the state of a degenerate Fermi gas), is produced by weak interactions of pairs of fermions with equal and opposite momenta. The interaction leading to the superfluidity of nuclear matter may also be regarded as weak. Indeed, the successful application of the shell model of the nucleus indicates that, in a first approximation, the forces of interaction between nucleons can be reduced to a certain effective potential that does not affect the appearance of the superfluid state, and to a change in the nucleon mass. Moreover, the interaction between nucleons that leads to the superfluid state depends on the degree of ordering in the motion of the nucleons, which may be small.

The purpose of the present work is to find the conditions that the proton-proton interaction potential must satisfy in order for the existence of a superfluid state of nuclear matter to be possible. We shall restrict our consideration to proton-proton interactions (assuming that the p - p interactions are equal to the n - n interactions) and shall not take neutron-proton interactions into account. Apparently, this case is of greatest interest in the transition from nuclear matter to a finite nucleus.

Let us consider only that part of the interaction Hamiltonian which is responsible for the appearance of the superfluid state of nuclear matter, i.e., the model Hamiltonian, which we write in the form

$$H = \sum_k \{E(k) - E_F\} a_{k+}^+ a_{k+} + \frac{1}{V} \sum_{k,k'} J(k, k') a_{k+}^+ a_{-k-}^+ a_{-k'-} a_{k'+}, \quad (1)$$

where V is the volume of the system; E_F is a parameter playing the role of the

chemical potential, and in the normal state it is equal to the energy of the Fermi surface; $a_{k\pm}^+$, $a_{k\pm}$ are the creation and annihilation operators of a proton, the signs \pm characterizing the direction of the spin. The function $J(k, k')$ is related to the singlet $J_s(k, k')$, triplet $J_t(k, k')$, and tensor $J_T(k, k')$ potential functions of the p - p interaction in the following way:

$$J(k, k') = J_s(k, k') + J_s(k, -k') + J_t(k, k') - J_t(k, -k') + 2(3e_z^2 - e^2) [J_T(k, k') - J_T(k, -k')], \quad (2)$$

where

$$\mathbf{e} = \frac{\mathbf{k} - \mathbf{k}'}{|\mathbf{k} - \mathbf{k}'|}.$$

Following the arguments of N. N. Bogolyubov's paper ⁽²⁾, we find that a superfluid state of nuclear matter exists in the case when there are solutions of the equation

$$2|E(k) - E_F|\Psi(k) + \frac{1}{V} \sum_{k'} J(k, k')\Psi(k') = E\Psi(k) \quad (3)$$

with negative eigenvalues $E = -2\delta$, $\delta > 0$.

Let us investigate the asymptotic solutions of (3) as $J(k, k')$ tends to zero, when E also tends to zero while remaining negative. We put

$$\Psi(k) = \frac{\chi(k, \Omega)}{|E(k) - E_F| + \delta},$$

pass from summation to integration, and obtain

$$2\chi(k, \Omega) + \frac{1}{(2\pi)^3} \int d\Omega' \int_0^\infty k'^2 dk' \frac{J(|k|, |k'|, \cos \alpha)}{|E(k) - E_F| + \delta} \chi(k', \Omega') = 0, \quad (4)$$

where $\cos \alpha = \cos \vartheta \cos \vartheta' + \sin \vartheta \sin \vartheta' \cos(\varphi - \varphi')$.

Taking into account that, as $\delta \rightarrow 0$, the integral in (4) becomes logarithmically divergent near the Fermi surface, we proceed to consideration of the approximate equation

$$\chi(k, \Omega) + \ln \frac{mE'(k_F)}{\delta} \cdot \frac{k_F^2}{E'(k_F)} \frac{1}{(2\pi)^3} \int d\Omega' J(|k|, k_F, \cos \alpha) \chi(k_F, \Omega') -$$

$$-\frac{1}{2(2\pi)^3} \int d\Omega' \int_0^\infty dk' \ln |k' - k_F| \cdot \frac{d}{dk'} \left[J(|k|, |k'|, \cos \alpha) \frac{k'^2}{E'(k')} \chi(k', \Omega') \right] = 0, \quad (5)$$

which, for small $J(k, k')$, asymptotically coincides with (4).

We introduce a new function

$$f(k', \Omega') = \frac{\chi(k', \Omega')}{\chi(k_F, \Omega')} \cdot \frac{1}{\ln \frac{mE'(k_F)}{\delta}},$$

where

$$f(k_F) = \frac{1}{\ln \frac{mE'(k_F)}{\delta}}.$$

The equation for $f(k, \Omega)$ at the point $k = k_F$ is obtained in the form

$$f(k_F) + \frac{k_F^2}{E'(k_F)} \frac{1}{(2\pi)^3} \int d\Omega' J(k_F, k_F, \cos \alpha) \frac{\chi(k_F, \Omega')}{\chi(k_F, \Omega)} - \frac{1}{2(2\pi)^3} \int d\Omega' \int_0^\infty dk' \ln |k' - k_F| \cdot \frac{d}{dk'} \left[J(k_F, |k'|, \cos \alpha) \frac{k'^2}{E'(k')} f(k', \Omega') \frac{\chi(k_F, \Omega')}{\chi(k_F, \Omega)} \right] = 0. \quad (6)$$

As $J(k, k')$ tends to zero, equation (6) has a solution in the case if

$$\int d\Omega' \frac{\chi(k_F, \Omega')}{\chi(k_F, \Omega)} J(k_F, k_F, \cos \alpha) < 0. \quad (7)$$

Indeed, $f(k_F) > 0$, while the last term in (6) is a quantity of higher order of smallness, provided that for $J(k_F, |k'|, \cos \alpha)$ there is no region in k' -space where it changes rapidly.

Expanding the functions $J(k_F, k_F, \cos \alpha)$ and $\chi(k_F, \Omega)$ in series in Legendre polynomials, carrying out the corresponding integrations, we find the conditions for superfluidity of nuclear matter in the form

$$J_s^{l=0}(k_F) < 0, \quad J_s^{l=2}(k_F) < 0, \quad (8)$$

$$J_t^{l=1}(k_F) + 0.8, \quad J_T^{l=1}(k_F) < 0, \quad (9)$$

since each state with $l > 2$ gives a relatively small contribution to p - p scattering.

Thus, the appearance of a superfluid state of nuclear matter is connected with the behavior of the potential functions on the Fermi surface. For the possible existence of a superfluid state of nuclear matter it is necessary that, on the Fermi surface, the potentials be attractive potentials.

It is shown in ⁽³⁾ that the potentials of the proton-proton interaction have a repulsive character at wavelengths of order $(0.2-0.5) \cdot 10^{-13}$ cm and smaller, while at larger wavelengths they have an attractive character. The wavelength corresponding to the Fermi-surface energy is $\lambda = 0.675 \cdot 10^{-13}$ cm; therefore the p - p potentials near the Fermi surface have, for the most part, an attractive character.

Thus, on the basis of the data on the p - p potentials one may conclude: the conditions for superfluidity of nuclear matter are fulfilled if the p - p interactions play the main role in the appearance of the superfluid state and if these interactions are weak.

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CITED LITERATURE

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