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## Abstract

## Full Text

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# ON THE ABSORPTION OF SOUND OF FINITE AMPLITUDE IN A RELAXING MEDIUM

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When sound of finite amplitude propagates in any medium, a wave that is initially sinusoidal becomes distorted as it propagates and ultimately assumes a sawtooth form ([1]). As has been shown in a number of works, such a wave is absorbed much more strongly than a wave of infinitesimally small amplitude, and the absorption coefficient in this case depends on its amplitude.

In the present work we investigate the absorption of a sound wave of finite amplitude in a medium where internal processes are possible that are associated with the transfer of energy of the translational motion of molecules to internal degrees of freedom. These processes lead to relaxation and, consequently, to additional absorption of sound whose frequency is close to the relaxation frequency.

Usually a wave of finite amplitude is described by the Riemann solution ([1]). However, this solution is inapplicable where the wave becomes sawtooth-shaped and discontinuities arise. To describe such a wave we shall use the solution obtained in Fay' s work ([2]). The oscillatory velocity is equal to

$$u = \frac{2M\omega}{c_0(\gamma+1)} \sum_{n=1}^{\infty} \frac{\sin n(\omega t - kx)}{\operatorname{sh} n(\alpha_0 + \alpha x)}. \quad (1)$$

Here $M = \frac{4\eta_1}{3\rho_0}$; $\eta_1$ is the coefficient of shear viscosity; $\alpha = \frac{1}{2}\frac{M\omega^2}{c_0^3}$; $\alpha_0$ is a constant uniquely associated with the Reynolds number; $c_0$ is the speed of sound at equilibrium.

This expression gives the Fourier expansion for a sawtooth wave in a viscous medium. It was obtained as an approximate solution of the exact hydrodynamic equations with viscosity, the first approximation in $u/c_0$ being taken. The method of solution used in ([2]) makes it possible to take nonlinear effects into account already in the first approximation. The point $x = 0$ in (1) corresponds to a certain point in space at which a stabilized sawtooth wave, smoothed by the influence of shear viscosity, is specified. On the other hand, since expression (1) has been obtained in the first approximation, one may assume that the absorption of such a wave can be described by the relaxation theory developed for sound of infinitesimally small amplitudes.

On the basis of the thermodynamics of irreversible processes it can be shown that the energy dissipated in an irreversible process caused by a deviation from equilibrium is equal to

$$W = \frac{1}{L}\left(\frac{\xi}{\tau}\right)^2, \tag{2}$$

where $\xi = \zeta - \zeta_0$; $\zeta = n_2/n$ is the relative concentration of excited particles; $\zeta_0(\rho)$ is its equilibrium value; $n_2$ and $n_1$ are, respectively, the numbers of excited and unexcited particles per unit volume; $n = n_1 + n_2$; $\tau$ is the relaxation time; $L$ is a constant coefficient.

The reaction equation for determining $\xi$ has the form (see, for example, ([3]); temperature changes are not taken into account)

$$\frac{d\xi}{dt} = -\frac{\xi}{\tau} - \frac{\partial\zeta_0}{\partial\rho}\frac{\partial\rho}{\partial t}. \tag{3}$$

The solution of (3), taking into account the continuity equation in the linear approximation, is

$$\xi = \rho_0 e^{-t/\tau}\int_0^t e^{t/\tau}\frac{\partial\zeta_0}{\partial\rho}\frac{\partial u}{\partial x}\,dt. \tag{4}$$

If the mass of the particles does not change upon excitation (for example, rotational or vibrational degrees of freedom are excited), then $\zeta_0 = \rho_{20}/\rho$, where $\rho_{20}$ is the equilibrium density of the excited particles, and $\partial\zeta_0/\partial\rho = -\rho_{20}/\rho^2$. Substituting this expression and (1) into (4), we obtain

$$\xi = \frac{2M\omega^2\rho_{20}\tau}{\rho_0 c_0^2(1+\gamma)}\sum_{n=1}^{\infty}\frac{n\cos n(\omega t - kx) + n^2\omega\tau\sin n(\omega t - kx)}{[1+(n\omega\tau)^2]\,\mathrm{sh}\,n(\alpha_0 + \alpha x)}. \tag{5}$$

Substituting (5) into (2) and averaging over time, we obtain the mean dissipated energy

$$W = \frac{1}{2L}\left(\frac{\rho_{20}\cdot 2M\omega^2}{\rho_0 c_0^2(\gamma+1)}\right)^2\sum_{n=1}^{\infty}\frac{n^2}{[1+(n\omega\tau)^2]\,\mathrm{sh}^2\,n(\alpha_0+\alpha x)}. \tag{6}$$

We shall define the absorption coefficient $\varkappa$ as the ratio of the dissipated energy to twice the energy flux (1), which here is equal to $pu$, where $p$ is the variable pressure. We determine the coefficient $L$ by finding $W$ for a sinusoidal wave and comparing the result obtained with the known Mandelstam and Leontovich formula for the absorption coefficient ([4,1]). Finally:

$$\varkappa = \frac{1}{2c_0}\left[\left(\frac{c_\infty}{c_0}\right)^2 - 1\right]\omega^2\tau\frac{\sum_{n=1}^{\infty}\frac{n^2}{[1+(n\omega\tau)^2]\,\mathrm{sh}^2\, n(\alpha_0+\alpha x)}}{\sum_{n=1}^{\infty}\frac{1}{\mathrm{sh}^2\, n(\alpha_0+\alpha x)}}. \qquad (7)$$

We shall determine the quantity $\alpha_0$ in the following way. If, on the basis of (1), one writes the expression for the variable pressure $p$ ([2]), then the amplitude of the first harmonic $p_1$ in the spectral expansion of the wave at the point $x = 0$ is equal to

$$p_1 = \frac{P_0\gamma}{1+\gamma}\frac{2M\omega}{c_0^2\,\mathrm{sh}\,\alpha_0}.$$

Let us introduce the parameter $R = p_1/\eta_1\omega$, analogous to the Reynolds number (the Reynolds number is usually defined as $p/\eta_1\omega$, where, in contrast to our case, $p$ is the amplitude of the entire wave). Then it is clear that, for each medium, there is a unique relation between $R$ and $\alpha_0$.

Figure 1 shows the dependence of the quantity

$$K = \frac{2c_0\varkappa}{[(c_\infty/c_0)^2 - 1]\,\omega} \qquad (8)$$

on $\omega\tau$ for $CO_2$ ($\gamma = 1.3$; $c_0 = 259$ m/sec, relaxation frequency $\omega_p = 2\cdot 10^5$ Hz). Curve $a$ is plotted for sound of infinitely small amplitude; the absorption maximum is at the frequency $\omega_p = 1/\tau$. Curves $b$ and $c$ are plotted for values of the parameter $R$ equal respectively to 4.3 and 50. In the calculations $x = 0$ was assumed.

It is seen from the graph that, when $R$ is increased by approximately a factor of 10, the absorption at the maximum increases by approximately a factor of 1.5. In addition, as $R$ increases, the absorption maximum shifts toward frequencies lower than the relaxation frequency. Physically such a result is quite understandable. Nonlinear distortions are equivalent to the appearance in the spectrum of the wave of higher harmonics, each of which receives part of the energy being transported. Consequently, if the fundamental sound frequency is less than the relaxation frequency,

**Fig. 1**

**Fig. 2**

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

$\omega < 1/\tau$, then the higher harmonics fall into the relaxation region, which leads to an increase in absorption.

Figure 2 gives the dependence of the value $K$ at the maximum on the parameter $R$. It is seen that the absorption coefficient increases with increasing $R$. This agrees with the increase in absorption of finite-amplitude waves as a result of the change in their shape, as has been noted in a number of works.

Let us note that the indicated effect is rather significant in magnitude and can be detected experimentally. However, in setting up the experiment one should choose a medium in which the absorption is not very large, i.e., the Reynolds number $\gg 1$. Otherwise the wave will be attenuated before the nonlinear distortions lead to the formation of a sawtooth wave.

It should also be noted that, if $p_1$ is kept unchanged, then the absorption in a certain frequency range will practically not depend on frequency.

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*Note: Figure translations are in progress. See original paper for figures.*