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Fig. 1. Schematic diagram of a regenerative oscillator

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Abstract

Full Text

PHYSICS

I. F. Shchegolev

SIGNAL-TO-NOISE RATIO IN A REGENERATIVE DETECTOR OF NUCLEAR PARAMAGNETIC RESONANCE

(Presented by Academician P. L. Kapitsa on 16 June 1958)

When observing nuclear paramagnetic resonance, as a rule, the signal-to-noise ratio is small, with the exception of the case of proton resonance. But even in this latter case it is often necessary to observe the signal in a small amount of substance, in weak or inhomogeneous fields, and all this noticeably reduces the signal-to-noise ratio.

Usually, in estimating the signal-to-noise ratio when choosing the parameters of a regenerative-detector circuit, one uses the expression obtained by Blombergen, Purcell, and Pound ([1]), who analyzed the operation of a bridge circuit (see, for example, ([2])). It is assumed that the presence of a regenerative tube can be taken into account by introducing a certain noise factor, which is not very large at low levels of generation, so that the entire expression remains approximately valid.

Fig. 1. Schematic diagram of a regenerative oscillator

The purpose of the present note is to show that in fact the principal noise sources in these two cases have a completely different character: whereas in the bridge circuit these are thermal noises in the tank circuit, in the regenerative circuit the principal source of noise is the relatively slow fluctuations of the tube transconductance, arising from such effects as the flicker effect, etc.

For the subsequent analysis one may consider the simplest oscillator circuit shown in Fig. 1. If the tube characteristic is approximated by a third-degree polynomial and the reaction of the anode is neglected, then one may write

$$i_a = i_0 + G_1 v - {}^4/_3 G_3 v^3. \qquad (1)$$

Here v is the voltage on the oscillatory circuit; i_0 is the constant component of the anode current; $G_1 = \chi S$, where χ is the feedback coefficient and S is

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the transconductance of the tube. The coefficient G_3 takes into account the nonlinearity of the tube characteristic.

To observe a nuclear-resonance signal, the specimen under investigation is placed in the tank coil, and the coil with the specimen is placed in a constant magnetic field H_0. When the generation frequency ω approaches the resonance frequency $\omega_0 = \gamma H_0$, where γ is the nuclear gyromagnetic ratio, additional energy absorption arises in the specimen as a result of nuclear resonance, and the amplitude of generation decreases. If the field H_0 is modulated with a small frequency Ω, then the voltage on the circuit becomes modulated:

$$v = (U_0 + U_\omega \sin \Omega t) \sin \omega t$$

(we take into account only the fundamental harmonic of the modulating frequency).

U'_{c0} is the nuclear-resonance signal that is to be measured by the detector output instrument. Its magnitude depends, in particular, on the closeness of the frequency ω to the resonance value ω_0, and was calculated in ([3]). If the modulation amplitude ΔH_0 does not exceed the linewidth, and the modulation frequency $\Omega \ll p = (G_1 - G_\text{k})/C$, then, using the Bloch expressions for the dynamic nuclear susceptibility, one may write that the maximum value of the effective signal amplitude is

$$U_\text{c}^* = \frac{\pi A}{2\sqrt{2}} \frac{\chi_0 T_2 C \omega_0^2 \xi}{G_3 U_0 \left[1 + (\pi C / V_\text{k}) U_0^2 \gamma^2 T_1 T_2\right]^{3/2}}. \tag{2}$$

Here χ_0 is the static nuclear susceptibility; T_1, T_2 are, respectively, the longitudinal and transverse relaxation times; C is the capacitance of the circuit; V_k is the volume of the coil; ξ is the fraction of the coil volume occupied by the sample. The quantity $A = \gamma T_2 \Delta H_0 \ll 1$.

The presence in the circuit of various noise sources causes fluctuations of the oscillation amplitude, which constitute the noise against the background of which the nuclear-resonance signal is observed. To calculate the spectral density of these amplitude fluctuations, let us write the equations describing the self-oscillating system (Fig. 1):

$$L \frac{di}{dt} = v; \tag{3}$$

$$C \frac{dv}{dt} = -i - vG_\text{k} + i_\text{a} + i_\text{t}. \tag{4}$$

Here i_t takes into account the thermal noise arising in the equivalent resistance of the circuit. We shall take the tube noise into account if we assume that

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the constant component of the anode current and the tube transconductance undergo random fluctuations about their mean values, which arise because of nonuniform emission, mechanical vibrations, and other causes.

If we put $v = V\sin(\omega t + \vartheta)$, then from equations (3) and (4) one can obtain in the usual way ([4])

$$C\frac{dV}{dt} = -G_{\mathrm{k}}V\sin^2(\omega t + \vartheta) + (i_{\mathrm{a}} + i_{\mathrm{t}})\sin(\omega t + \vartheta). \tag{5}$$

Let us introduce the notation: $V = U_0 + u,\ G_1 = G + g,\ i_0 = I_0 + i_{\mathrm{sh}}$, where U_0, G, I_0 are the mean values of the corresponding quantities, and u, g, i_{sh} are small fluctuations about the mean values. Then from equation (5) we obtain the equation for the amplitude fluctuations

$$\frac{du}{dt} + pu - \frac{3}{2}pu\cos 2(\omega t + \vartheta) + \frac{1}{2}pu\cos 4(\omega t + \vartheta) =$$

$$= \frac{U_0}{2C}g + \frac{i_{\mathrm{sh}} + i_{\mathrm{t}}}{C}\sin(\omega t + \vartheta) - \frac{U_0}{2C}g\cos 2(\omega t + \vartheta). \tag{6}$$

Here $p = (G - G_{\mathrm{k}})/C$ is the stability parameter of the self-oscillating system. Since we are interested in the noise components near the modulation frequency Ω, and in deriving formula (2) it was assumed that $\Omega \ll p$, we shall neglect, on the left-hand side of equation (6), the quantity du/dt in comparison with pu. Let us note that, because of the selective action of the circuit, only such low-frequency noise components i will be significant. After this, let us write the same equality (6) for the time instant $t+\tau$, then multiply, respectively, their right- and left-hand sides and statistically average. The correlation function $\overline{u(t)u(t+\tau)}$ obtained in this way depends explicitly on time. In order to obtain the spectral distribution of the noise, it is necessary to average the entire expression over time. Thus we find an expression for

of the function $\psi_u(\tau) = \overline{\overline{u(t)u(t+\tau)}}^{\,t}$. The spectral density is

$$F_u(\Omega) = \frac{2}{\pi}\int_0^\infty \psi_u(\tau)\cos\Omega\tau\,d\tau.$$

In carrying out the integration, we shall take into account that tube noise is uniform at high frequencies (of order ω) and increases strongly in the low-frequency region (of order Ω). We obtain

$$U_{\mathrm{n}} = \sqrt{F_u(\Omega)\Delta\Omega} = \frac{\sqrt{\Delta\Omega}}{2G_3}\sqrt{\frac{2F_i(\omega) + 2F_T(\omega)}{U_0^4} + \frac{x^2F_s(\Omega)}{U_0^2}}. \tag{7}$$

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Fig. 2. Dependence of the noise voltage on the oscillation amplitude (U_{n} in arbitrary units)

Figure 2: Fig. 2. Dependence of the noise voltage on the oscillation amplitude (U_{n} in arbitrary units)

Fig. 3. Spectral distribution of low-frequency fluctuations of the oscillation amplitude ($F_u(\Omega)$ in arbitrary units)

Figure 3: Fig. 3. Spectral distribution of low-frequency fluctuations of the oscillation amplitude ($F_u(\Omega)$ in arbitrary units)

Here $F_i(\omega)$ is the spectral density of fluctuations of the dc component of the anode current at frequency ω; $F_T(\omega)$ is the spectral density of thermal current fluctuations in the equivalent resistance of the circuit; $F_s(\Omega)$ is the spectral density of fluctuations of the tube transconductance at frequency Ω.

Fig. 2. Dependence of the noise voltage on the oscillation amplitude (U_{n} in arbitrary units)

To determine which term under the radical plays the principal role, the dependence of the noise voltage on the generator circuit on the oscillation amplitude was recorded. The generator was assembled with a 6N15P dual triode ([5]), with the second grid grounded. The oscillation amplitude could be varied without changing the tube parameters by changing the circuit capacitance or the value of the shunt resistance of the circuit. The voltage taken from the generator circuit was amplified at high frequency and then detected. The noise voltage obtained at the detector load, caused by fluctuations of the oscillation amplitude, after preliminary amplification by a narrow-band low-frequency amplifier was measured with a thermal instrument. Calibration of the circuit consisted in applying to the input of the high-frequency amplifier a modulated voltage from a GSS-6 generator and recording the dependence of the voltage at the output of the low-frequency amplifier on the amplitude of the high-frequency voltage. The modulation amplitude was chosen so small that its further decrease no longer changed the results. This calibration made it possible, from the output noise voltage, to determine the noise voltage that modulates the oscillation amplitude in the circuit.

Fig. 3. Spectral distribution of low-frequency fluctuations of the oscillation amplitude ($F_u(\Omega)$ in arbitrary units)

The results are given in Fig. 2. It is seen that the noise voltage is inversely proportional to the oscillation amplitude. Thus, the principal source of noise in a regenerative detector is noise arising from slow fluctuations of the tube transconductance. The sharp increase of the noise with decreasing frequency, shown in Fig. 3, is determined by the corresponding behavior of the function $F_s(\Omega)$.

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Fig. 4. Dependence of the signal-to-noise ratio on the circuit capacitance

Figure 4: Fig. 4. Dependence of the signal-to-noise ratio on the circuit capacitance

Taking the foregoing into account, from formulas (2) and (7) we obtain:

$$k = \frac{U_c}{U_{\text{sh}}} = \frac{\pi A}{\sqrt{2}} \frac{\chi_0 T_2 C \omega_0^2 \xi}{\chi \left[1 + (\pi C / V_\kappa) U_0^2 \gamma^2 T_1 T_2\right]^{1/2} \sqrt{F_s(\Omega)\, \Delta\Omega}}. \tag{8}$$

The signal-to-noise ratio increases as the oscillation amplitude is decreased and, at sufficiently small amplitudes, ceases to depend on it. Further, there is no direct dependence of k on the quality factor of the circuit, Q. Other conditions being equal, in order to increase k, one must reduce the quality factor, since this reduces U_0. However, reducing the oscillation amplitude by reducing the quality factor of the circuit (or by reducing the tube transconductance) is not the best way to increase the signal-to-noise ratio. It is much more advantageous to reduce the oscillation amplitude by increasing the circuit capacitance C or decreasing the feedback coefficient χ, since this leads to a marked increase in k.

Fig. 4. Dependence of the signal-to-noise ratio on the circuit capacitance

Figure 4 shows the dependence of the signal-to-noise ratio on the circuit capacitance at a constant oscillation amplitude. The measurements were carried out in one and the same field H_0 on the protons of water, in which ferric nitrate was dissolved in order to reduce the relaxation times. The product $T_1 T_2$ was $5 \cdot 10^{-9}$ sec^2, and the nuclear resonance was far from saturation even at the largest capacitances.

In those cases when the relaxation times are large, the signal-to-noise ratio can be increased by decreasing the feedback coefficient. The maximum value of the quantity C/χ is determined by the requirement of stability of the oscillation amplitude: $p = (G - G_k)/C > 0$. Hence we obtain that $(C/\chi)_{\max} = SQ/\omega_0$. Thus, in order to obtain the maximum value of the signal-to-noise ratio, one should, as far as possible, increase the tube transconductance and the quality factor of the circuit so that operation is possible at the largest possible values of C/χ.

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Institute of Physical Problems
Academy of Sciences of the USSR

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