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Abstract

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GEOPHYSICS

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CHARACTERISTIC PARAMETERS OF THE FIELD OF WIND WAVES IN THE OCEAN

The results of new measurements of waves in the ocean, compared with the results of experiments in the storm basin of the Academy of Sciences of the USSR, make it possible to characterize the field of storm waves far beyond the initial stages of their development. It has become possible to determine the principal parameters of the field, to find scales for the use of our dimensionless functions and arguments ([1]), and approximately to take into account the variability of these scales.

To solve the problem posed, let us recall that the transfer of wind energy to waves is characterized $((^1), pp.78 and 112)$ by the aerodynamic coefficient $\bar{\chi}$, which depends on the steepness of the waves: $\bar{\chi} = \varkappa r/R$. Here r is the radius of the orbit of surface particles on the wave, and R is the so-called radius of the rolling circle $(R = \lambda/2\pi)$. In turn, the dimensionless coefficient $\varkappa$ enters into the expression for the limiting value r_∞ for storm waves in the ocean at a given wind speed V:

$$r_\infty = \frac{9}{\pi}\frac{\varkappa}{k^2}\frac{\delta_a}{\delta}\left(\frac{R}{r}\right)_\infty \frac{(V-c)^2}{g}. \tag{1}$$

Here δ_a and δ are the densities of air and water; c is the phase velocity of the waves; g is the acceleration in the gravitational field; k is a coefficient characterizing turbulent phenomena in the water during waves. Introducing the coefficients $\varkappa$ and k into the complete balance equation for wind waves at an arbitrary distance x from the windward shore. For an arbitrary moment of time, separated from the beginning of the wind action by an interval t, we write

$$\delta g r\frac{\partial r}{\partial t} = \varkappa\left(\frac{r}{R}\right)\frac{2r}{T}\delta_a(V-c)^2 - \frac{2\pi}{9}\delta g\frac{k^2}{T}\left(\frac{r}{R}\right)^2 r^2 - \frac{5}{8}\delta g^{3/2}\left(\frac{R}{r}\right)^{1/2} r^{3/2}\frac{\partial r}{\partial x}. \tag{2}$$

Dividing both sides of equation (2) by δgr and using expression (1), borrowed from work ([1]), we again present the energy-balance equation in its simplest form, passing from the arguments x, t to the dimensionless arguments ξ, τ and from the function r to the dimensionless function η:

$$\frac{\partial \eta}{\partial \tau} = 1 - \eta - \eta^{1/2} \frac{\partial \eta}{\partial \xi}. \tag{3}$$

We shall now retain the former form only for the expression $\eta = r/r_\infty$ (([1]), p.95). As for the expressions for ξ and τ, they will be written in a new form, which makes it possible to determine the length scales and time scales better. First we write the new expressions as applied to the final stage of waves—to the established wave state—and then we shall try to take into account changes of the scales at intermediate stages. Then it will turn out, first of all:

$$\frac{x}{\xi} = 0.895\, g^{1/2} \frac{T_\infty^{5/2}}{k^2} \left(\frac{R}{r}\right)_\infty^{5/2} r_\infty^{1/2}. \tag{4}$$

But it is known that, on the one hand, $g^{-1/2} D_\infty^{1/2} = c_\infty$. On the other hand, c_∞ is expressed in terms of V by the relation derived in ([1]), § 24: $c_\infty / V = f_\infty$. Consequently, one may write:

$$\frac{x}{\xi} = 0.895 f_\infty \frac{T}{k^2} \left(\frac{R}{r}\right)^2 V. \tag{5}$$

On the basis of a large number of instrumental measurements it may be considered, with sufficient reliability, that $f_\infty = 0.75$; $(R/r)_\infty = 8$ (this value also follows from our theory ([2], [3])). Substituting these numbers into (5), we obtain an expression for the important coefficient k in explicit form:

$$k = 6.56 \sqrt{\frac{\xi}{x} V T}. \tag{6}$$

We were able to determine the numerical value of k on the basis of observations of practically established waves approaching the ship from the windward shore of a deep sea. The initial quantities were $x = 104$ km, $V = 9.7$ m/sec, $T = 4.2$ sec. The measured wave period T corresponds to the wave length $\lambda = 27.5$ m. Recent records of wave profiles in a storm basin make it possible to assume that the greatest possible wave steepness is attained at $\lambda_0 \approx 1$ m. Consequently, by formula (22) of our paper ([2]), one may put $h = 1.4$ m. From the curve of Fig. 46 of the monograph ([1]) we find, for the given wind velocity, the limiting possible wave height in the ocean $h_\infty = 2$ m. Consequently, $\eta = 1.4/2 = 0.7$.

Fig. 1

From the curve of Fig. 38 of the monograph ([1]) we determine, for this value of η, the corresponding value $\xi = 0.76$. In accordance with this, on the basis of

Fig. 1

Figure 1: Fig. 1

equation (6), $k = 0.113$. Substituting this value together with the other known numbers into formula (5) to calculate the scale interval, we obtain

$$\frac{x}{\xi} = 3360VT \text{ m} = 3.36VT \text{ km}. \tag{7}$$

The numerical value found for k differs from the number ($k = 0.36$) which was obtained by Karman from experiments on turbulent flows in pipes ([1], p. 92; [4]). Therefore let us check the calculations using the example of enormous waves that were observed on the Black Sea during an exceptionally long storm in January 1931. At that time a wind with a velocity of 22 m/sec blew for 2 days in succession in the WSW direction and produced storm waves 170 m long in the eastern part of the sea at $x = 1100$ km. On the basis of [1], according to the curve of Fig. 46, this wind velocity corresponds to $h_\infty = 9$ m, and on the basis of [2] and the approximate assumption $\lambda_0 \approx 1$ m, $h_0 \approx 1/7$ m, it should be assumed that approximately $\lambda_\infty = 200$ m and $T_\infty = 11.3$ sec. Then, by (7), we obtain $x/\xi = 870$ km, and hence the greatest dimensionless distance is $\xi = 1100/870 = 1.27$. According to the curve of Fig. 38 of the monograph [1], this value corresponds to $\eta = 0.835$. Consequently, one may expect the formation of established waves with height $h = 7.5$ m in the first approximation.

In Fig. 1, curve 1 gives, in the first approximation, the distribution of wave heights at different distances x from the windward shore, analogous to curve a in Fig. 38 of the monograph [1], for the value of the scale interval $x/\xi = 870$ km.

We shall now find the second approximation, taking into account the variability of the scales in the field of storm waves. Unfortunately, it is impossible to take into account separately the variability of the coefficient x and of the quantity $V - c$. Trial calculations suggest that, at practically important stages of wave development, the product $x(V-c)$ differs little from $x_\infty(V-c)_\infty$: the decrease of the second factor is compensated by the increase of the first. Therefore, for the moment we shall confine ourselves to allowing for the change of the quantities T and $(R/r)^2$, which affect the scales. First of all, it can be shown that the corrected values of the abscissas of the diagrams in Fig. 1 are equal to the product of the abscissas of curve 1 by

$$\frac{T}{T_\infty}\frac{(R/r)^2}{(R/r)^2_\infty}.$$

Fig. 2

In turn,

$$\frac{T}{T_\infty}\frac{(R/r)^2}{(R/r)^2_\infty} = \left(\frac{\lambda}{\lambda_\infty}\right)^{1/2}\frac{(R/r)^2}{(R/r)^2_\infty} = \left(\frac{h}{h_\infty}\right)^{1/2}\frac{(R/r)^{5/2}}{(R/r)^{5/2}_\infty}.$$

On the basis of work (2) and on the basis of the approximate value $\lambda_0 \approx 1$ m, Fig. 2 gives the curve representing

$$\zeta = \frac{(R/r)^{5/2}}{(R/r)^{5/2}_\infty}$$

as a function of the wave height h. On the other hand, the fraction $(h/h_\infty)^{1/2}$ is determined directly for each value of the argument h, since the value h_∞ corresponding to the given wind speed V is known. After multiplying the abscissas of curve 1 by $\zeta(h/h_\infty)^{1/2}$, the points were obtained through which curve 2 in Fig. 1 was drawn. This is the second approximation to the true distribution of the heights of established waves.

The growth of waves in time at an infinitely large distance from the windward shore could be described by a simple formula obtained by integrating (3) under the condition $\partial\eta/\partial\xi = 0$. Namely,

$$\eta = 1 - e^{-\tau}. \tag{8}$$

The time scale in the first approximation is determined by comparing (2) with (3) at the final stage of wave development:

$$\frac{t}{\tau} = \frac{9}{2\pi}\left(\frac{R}{r}\right)^2\frac{T}{k^2}. \tag{9}$$

Substituting into (9) the numerical value of k found and the above-mentioned value $(R/r)_\infty$, we obtain, instead of (9), the working formula $t/\tau = 7800\,T$ sec. $= 3.36\,T$ hr. For the first approximation one may assume* on this basis and on the basis of (8) that, at a point located a distance x from the windward shore, the height $h_{x,t}$ of unestablished waves is expressed in terms of the height of established waves $h_{x,\infty}$ (at the same place) by the simple formula

$$h_{x,t} = h_{x,\infty}\left(1 - e^{-t/2.16T}\right); \tag{10}$$

here the wave period is expressed in seconds. As noted, $T_\infty = 11.3$ sec. Hence the time scale will be $t/\tau = 24.4$ hr. It is precisely this quantity that appears in the denominator of the fraction in the exponent of e in (10). According to Fig. 1, in the second approximation the height of established waves at a distance of 1100 m from the windward shore is 8.1 m. Hence, in (10) one must substitute

Fig. 3

Figure 2: Fig. 3

$h_{x,\infty} = 8.1$. As a result, by (10) we obtain curve 1 in Fig. 3, giving, in the first approximation, the law of wave growth near the leeward shore.

* In work (1) it is shown that this is one of the approximations to the complete integral of equation (3) of the present article, and moreover the most convenient one for taking into account changes of scales.

We find the second approximation by the same method by which the second approximation was found in the diagram of Fig. 1: the structure of formula (9) shows that the changes in T and $(R/r)^2$ at the intermediate stages can still be taken into account by multiplying the abscissae of curve 1 by the function ζ, determined from the diagram in Fig. 2, and by $(h/h_\infty)^{1/2}$.

In Fig. 3 the points obtained in this way are plotted, and curve 2 has been drawn through them; it gives the law of increase of heights near a leeward shore in the second approximation. As we see, even after a long duration of the storm—2 days—the waves cannot be considered established: they could still have increased in height by no less than 0.6 m.

Fig. 3

The numerical result itself of the analysis of the wind-wave field, carried out on the basis of the value found for the parameter k, has led us to somewhat underestimated magnitudes: the maximum wave height was not 7.5 m, but, in all probability, about 7.8-8 m.

At present one cannot claim great accuracy not only in the analysis of the wave field, but also in the very determination of the coefficient k: such a determination must be made on the basis of a larger number of reliable instrumental measurements of waves in the ocean. Such material will undoubtedly be available to investigators as a result of the work of the 3rd International Geophysical Year. In any case, it may already be asserted that the sole hypothesis introduced above by us for correcting the scales does not lead to gross errors in the analysis of the field of storm waves. The remaining aspects of the analysis are free from shaky empiricism and from any special assumptions whatsoever.

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Note: Figure translations are in progress. See original paper for figures.