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Abstract

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A NOTE CONCERNING DISPERSION RELATIONS FOR THE SCATTERING OF π-MESONS BY NUCLEONS

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As is well known, the basis for deriving dispersion relations is the causality condition in one form or another. In considering the processes of scattering of π-mesons by nucleons ([1]), it is usually assumed that the π-meson is an elementary particle whose behavior at infinity is described by creation and annihilation operators of a pseudoscalar field. It has been asserted that such an assumption is not only sufficient, but also necessary for deriving dispersion relations. Below it will be shown that this is not so. Namely, we shall assume that the elementary particle satisfying the locality condition is the nucleon, while with respect to the π-meson we shall make no such assumptions, i.e., by a π-meson we shall understand a certain bound complex in the sense of ([1]). Having adopted this assumption, we shall prove that it is sufficient for obtaining dispersion relations.

In accordance with what has been said, let us consider the transition amplitude

$$S(\omega, p; \omega', p') = (2\pi)^3 \langle \Phi^+_{q',\rho'} a^+_{s'}(p') S a_s(p) \Phi_{q,\rho} \rangle,$$

where a and a^+ are the nucleon annihilation and creation operators. Define the spinor currents by the relations

$$j(x) = i\frac{\delta S}{\delta \overline{\Psi}(x)} S^+; \qquad \bar{j}(x) = i\frac{\delta S}{\delta \Psi(x)} S^+.$$

Then, with the aid of the usual procedure, using the technique of variation with respect to the spinor field (see, for example, ([1])), we find

$$S(\omega, p; \omega', p') =$$

$$= i(2\pi)^4 \delta(q' + p' - q - p)\, \overline{u}_{\nu'}(p)\, M\left(q, \rho, i; q', \rho', i'; \frac{p' + p}{2}\right) \gamma^0 u_\nu(p),$$

where

$$M\left(q,\rho,i;q',\rho',i';\frac{p'+p}{2}\right)=$$

$$=i\int\exp\left[-i\frac{p'+p}{2}x\right]\theta(x^0)\left\{F(q,\rho,i;q',\rho',i';x)+\mathfrak{P}F(q,\rho,i;q',\rho',i';-x)\right\}dx,$$

$$F_{\alpha,\beta}(q,\rho,i;q',\rho',i';x)=\frac{1}{(2\pi)^3}\sum_{(n)}\int dl\,\langle\Psi_{q',\rho'}J_{\alpha,i}(0)\Psi_{l,n}\rangle\langle\Psi_{l,n}J_{\beta,i}(0)\Psi_{q,\rho}\rangle\times$$

$$\times\exp\left[-i\left(q_n^0-\frac{q'^0+q^0}{2}\right)x^0+i\left(\mathbf{l}-\frac{\mathbf{q}'+\mathbf{q}}{2}\right)\mathbf{x}\right].$$

Put $M=D'+iA'$; then it turns out that in the frame of reference $\mathbf{q}+\mathbf{q}'=0$, in which $\mathbf{p}+\mathbf{p}'=2\lambda\mathbf{e}$, $\mathbf{q}\cdot\mathbf{e}=0$, $p^0=E$, the quantities D' and A' will be connected by dispersion relations of the usual form, with integration over E from 0 to ∞. It is easy to see that the complete quantities $D=\bar{u}D'\gamma^0u$ and $A=\bar{u}A'\gamma^0u$ will be connected by the same dispersion relations. Let us consider, for example, the relation

$$\mathfrak{S}_e(1-\mathfrak{P})D(E)-\mathfrak{S}_e(1-\mathfrak{P})D(E_0)=$$

$$=\frac{\mathcal{P}}{\pi}\int_0^\infty 2E'\,\frac{E^2-E_0^2}{(E'^2-E^2)(E'^2-E_0^2)}\,\mathfrak{S}_e(1-\mathfrak{P})A(E')\,dE'. \qquad (1)$$

All the remaining relations are considered in a completely analogous way. To bring relation (1) to the usual form, it is sufficient to perform in it a Lorentz transformation from the introduced frame of reference to the Breit frame:

$$q\to Q=Lq,\qquad q'\to Q'=Lq',\qquad p\to P=Lp,\qquad p'\to P'=Lp',\quad (2)$$

where $\mathbf{P}+\mathbf{P}'=0$. It is easy to see that transformation (2) leads to the following change of variables:

$$\mathbf{q}=-\mathbf{P};\qquad \mathbf{Q}=\Lambda\mathbf{e}-\mathbf{P};\qquad \mathbf{Q}'=\Lambda\mathbf{e}+\mathbf{P};\qquad \Lambda=-\frac{\sqrt{\mu^2+\mathbf{q}^2}}{\sqrt{M^2+\mathbf{q}^2}}\lambda.$$

The integration variable E is replaced by the new variable $\mathcal{E}=Q^0$, with

$$E = \frac{\sqrt{M^2 + \mathbf{P}^2}}{\sqrt{\mu^2 + \mathbf{P}^2}}\,\mathcal{E}.$$

In addition, we have

$$S(Q, P, \rho, \nu, i; Q', P', \rho', \nu', i') =$$

$$= \sum_{(\nu'', \nu''')} \overset{*}{Z}_{\nu,\nu''}(L;p)\, Z_{\nu',\nu'''}(L;p')\, S(q, p, \rho, \nu'', i;\ q', p', \nu''', i').$$

The properties of the introduced matrix Z are simplest to establish if one considers the forward scattering amplitude, which is proportional to the Lorentz-invariant total effective cross section. In this way we find that $Z^{+}Z$ is proportional to the identity matrix, the coefficient of proportionality depending on the method of normalization of the spinors $u(p)$. The contribution from the bound state in relation (1) is found by means of the following expressions for the vertex parts:

$$\langle \Psi_{q',\rho'} J_{\alpha,i}(0) \Psi_{p,s''} \rangle = g\{\gamma^5 u_{\nu''}(p)\}_{\alpha} \tau^{\rho'}_{i,i'},$$

$$\langle \overset{*}{\Psi}_{p,s''} J_{\beta,i'}(0) \Psi_{q,\rho} \rangle = g\{\bar{u}_{\nu''}(p)\gamma^5\}_{\beta} \tau^{\rho}_{i'',i'}. \tag{3}$$

It is also easy to find the explicit form of the matrix Z corresponding to transformation (2). In view of its cumbersomeness, we do not give it.

Transforming relations (1) with the help of L and Z, and also taking (3) into account, we bring relation (1), after simple but somewhat cumbersome calculations, to the standard form. Thus it is fully proved that the assumption of the elementary nature of the π-meson is not obligatory for the derivation of the dispersion relations. The assumption of the elementary nature of the nucleon is also sufficient.

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CITED LITERATURE

1. N. N. Bogolyubov, D. V. Shirkov, *Introduction to the Theory of Quantized Fields*, Moscow, 1957.