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Abstract

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THE STEEPNESS OF DEVELOPING WAVES AT DIFFERENT WIND SPEEDS

At present we have theoretical schemes for the generation of waves on a smooth sea surface ([1]), for their growth up to the stage of greatest steepness ([2]), and a reliably verified theory of the development of waves from this stage to limiting dimensions in the ocean ([3]), with a continuous decrease in steepness down to the value $h/\lambda = 1/8\pi$.

Let us recall that, according to ([1]), newly generated waves have a finite length λ_1, depending on the wind speed and on the magnitude of the surface tension; according to ([2]), the steepness of the waves at first increases according to the law

$$\frac{h}{\lambda} = \frac{n}{2\pi}\left[1 - \left(\frac{\lambda_1}{\lambda}\right)^{2/3}\right] \tag{1}$$

(where n is a numerical coefficient, not yet known and apparently varying with time). This occurs owing to the transfer of part of the wind energy to the establishment of a definite velocity gradient of the drift current in the vertical. The increase in steepness ceases when waves of comparatively small length λ_2 reach the steepness $h/\lambda = 0.143$, the limiting value for kinematic reasons. The phenomena that caused the growth of steepness now contribute to a partial destruction of the crests of small waves, and, as the wavelength increases, their steepness remains unchanged ($h/\lambda = 0.143$) until the vertical gradient of current velocities corresponding to the given wind is established. Finally, after this gradient has been established, at some wavelength λ_0 the change in the angular momentum of the water particles in their orbits begins to obey the relations derived in ([3]), and the steepness of the developing waves decreases according to the law

$$\frac{h}{\lambda} = 0.04 + 0.103\left(\frac{\lambda_0}{\lambda}\right)^{2/3}. \tag{2}$$

In paper ([2]) an experimental curve is presented (Fig. 2) which essentially reproduces the course of the theoretical curve (Fig. 1). However, it is noted there that the experimental materials then available did not make it possible

Fig. 1

Figure 1: Fig. 1

to detect with sufficient clarity the breaks in the curve which, according to theoretical considerations, should be expected at the abscissa values $\lambda = \lambda_2$ and $\lambda = \lambda_0$. Moreover, in article ([2]) the question of the influence of wind speed on the course of curves of the type shown in Fig. 1 and Fig. 2 on p. 475 of the cited work was not touched upon at all.

Meanwhile, observations at sea, as well as observations in our storm basin, show that for one and the same wavelength the steepness of the waves is the greater, the greater the wind speed that generated them. The assumption naturally arises that only at high wind speeds does the intermediate stage of wave development in length from λ_2 to λ_0 prove significant; at lower wind speeds the segment $\lambda_0 - \lambda_2$ is shortened and at some definite wind speed may become zero; at still lower wind speeds the value $\lambda_0 = \lambda_2$ may shift still further to the side

of smaller wavelengths, and the descending part of the curve will begin before reaching the maximum ordinate of the ascending part; accordingly, the descending parts of the curve will lie the lower, the lower the wind speed that generated the waves.

The present work was carried out precisely in order to test such assumptions and to clarify the very interesting intermediate stage of wave development—from the wavelength λ_2 to the wavelength λ_0.

The experiments were conducted in the storm basin of the Marine Hydrophysical Institute of the Academy of Sciences of the USSR; moreover, in contrast to the usual method (4), the wave profile was photographed directly with a Kiev camera from a short distance (5 m) at short intervals (3-5 sec), and was not calculated from records of wave periods on a long strip of photographic paper. From such period records only the sections corresponding to wavelengths greater than 100 cm, which inspire complete confidence, were borrowed. Electric clocks and an electric stopwatch, which made it possible to record on each photograph the corresponding time with an accuracy of up to 0.01 sec, were in the field of view of the camera.

Fig. 1

For electrical engineering reasons it was impossible, in each series of experiments, to obtain an entirely exact wind speed in the basin specified in advance. Therefore all 33 completed series of photographs were divided into three groups corresponding to three wind speeds: $V = 7 \pm 1$ m/sec, $V = 11 \pm 1$ m/sec, and $V = 15 \pm 1$ m/sec.

Each time, from the moment the blower motors were started, the wind speed was accurately recorded by means of a self-recording micromanometer-anemograph designed by R. N. Ivanov. Unfortunately, a constant wind speed was established only a little before the point $\lambda = \lambda_2$, and therefore the ascending part of the curve of the change in wave steepness was not accessible for a sufficiently exact investigation at one or another prescribed constant speed. On the other hand, the most interesting—intermediate—stage of the curve from $\lambda = \lambda_2$ to λ_0 was obtained with sufficient reliability, despite the fact that the process of change in wave steepness at the initial stages of development proved to be extremely unsteady. At these initial stages, groups of waves having different heights and different steepness stand out very sharply. In photographing, we tried to record the most clearly developed waves in such groups; however, even in this way it was not possible to prevent the scatter of points in the diagrams at the initial stages of wave development at low and medium wind speeds.

In Fig. 1, wavelengths in centimeters are plotted along the abscissa axis. Here diagram A is constructed from experiments at a wind speed $V = 7 \pm 1$ m/sec; diagram B corresponds to a wind speed $V = 11 \pm 1$ m/sec

and diagram B—a wind speed $V = 15 \pm 1$ m/sec. As we see, the latter fully corresponds to the theoretical curve in Fig. 1 in paper ([2]). On it there is distinctly marked by points an interesting intermediate part, parallel to the axis of abscissas. The descending (bold) continuous curve, constructed according to theoretical formula (2) for the value $\lambda_0 = 85$, passes quite satisfactorily among the experimental points. As already mentioned, for technical reasons it is impossible to obtain in the tank an instantaneously established wind speed, and therefore the experimental points at the stage from $\lambda = \lambda_1$ to $\lambda = \lambda_2$ undoubtedly lie below those values of h/λ which would correspond to the full prescribed flow velocity. On the other hand, because of various inevitable disturbances (including those arising from not quite pure seawater in the large tank), it was impossible to expect the appearance of initial small waves in full correspondence with the delicate relations of work (1). As a result, it was necessary to restrict ourselves to constructing only an approximate dotted curve in formula (1), in the rising part of the characteristic. In doing so it was assumed that $\lambda_1 = 5$ cm, $\lambda_2 = 40$ cm. As we see, the experimental points here fell inside the curve.

Diagram of Fig. 1 shows that at wind speed $V = 11$ m/sec there is still preserved a small remnant of the intermediate stage of the curve from $\lambda = \lambda_2$ to $\lambda = \lambda_0$. The descending part of the theoretical curve agrees satisfactorily with the points obtained on the basis not only of new experiments, but also of numerous old records by the method of ([4]). We note that, in contrast to work ([4]), here we considered not the root-mean-square wave heights, but the heights of those waves that had developed most clearly, i.e. the greatest wave heights in each group. In view of the fact that the rising part of the curve has only a purely approximate character, the value λ_1 in the calculations by formula (1) is here kept the same as in diagram B. In accordance with the experimental points, the value λ_2 also remained as before. On the contrary, owing to the shortening of the length λ_0—λ_2, the value λ_0 in formula (2) here decreased considerably in comparison with diagram B: now $\lambda_0 = 45$ cm instead of 85 cm; this led to the descending curve in diagram running lower at all its points than the curve in diagram B.

Still lower went the theoretical (bold) curve in diagram A. Keeping the former approximate rising part of the curve, on this diagram we drew the descending curve with such a calculation that it should pass through quite reliable points corresponding to wavelengths greater than 100 cm. This curve corresponds to other initial conditions for the integration of the basic differential equation in ([3]) as compared with the curves of diagrams B and : the greatest wave steepness (0.122) here does not reach the limiting possible value 0.143. Taking as the initial condition $h/\lambda = 0.122$ at $\lambda_0 = 23$ cm, we obtained a new form of the integral for calculating the theoretical descending curve in diagram A:

$$\frac{h}{\lambda} = 0.04 + 0.082 \left(\frac{\lambda_0}{\lambda}\right)^{2/3}. \qquad (2')$$

For clarity, both this curve and the descending curve from diagram have been transferred to the lower diagram B (thin lines). As we see, in full agreement with our theoretical assumptions, as the speeds of the wind producing the wave decrease, the intermediate segment of length λ_0—λ_2, parallel to the axis of abscissas, is shortened. At speeds below 11 m/sec, this segment disappears altogether, and thereafter the descending part of the curve departs somewhere from the slope of the rising segment at the point with abscissa $\lambda_0 = \lambda_2$, as is seen, for example, in diagram A.

Considering the three descending curves combined in diagram C of Fig. 1, it is easy to notice that, at one and the same wavelength, the difference in their steepness turns out to be the greater, the smaller the corresponding wavelength.

As waves develop to the limiting dimensions possible in the ocean at a given wind speed, this difference steadily decreases and practically disappears in the later stages of wave development.

The laws found for the development of wind waves are of not only theoretical interest, illuminating subtle phenomena in the initial stages, but also of no lesser practical interest: knowing the descending portions of curves of types A, B, C, it is easy to improve the technique of scale correction discussed in detail in paper ([5]). Indeed, in the cited paper, waves in the ocean and in the deep sea were investigated at wind speeds above 20 m/sec, and the correction curve of Fig. 2 was calculated there as applied to the value $\lambda_0 = 100$ cm: starting from the theoretical formula (2) of the present paper, the correction factor ζ was determined, by which the abscissae of the first-approximation curves (in Figs. 1 and 3 of paper ([5])) had to be multiplied. In turn, the factor ζ may be expressed somewhat differently: not through the quantities R and r, but through the quantities λ and h, directly related to them. Let us recall that r is the half-height of the waves, and R is the radius of the rolling circle; the symbol ∞, placed below the corresponding quantities, means that the value is taken for

Fig. 2

Figure 2: Fig. 2

ocean conditions: $(R/r)_\infty = 8$ and $(h/\lambda)_\infty = 1/8\pi$. In this case the expression for ζ is written as follows:

Fig. 2

$$\zeta = \left[\frac{(h/\lambda)_\infty}{h/\lambda}\right]^{5/2}. \qquad (3)$$

Having computed, by formula (3), the values of ζ for different wavelengths, we found that at small and medium wind speeds the curves $\zeta(h)$ differ noticeably from the curve given in paper ([5]) in Fig. 2. At the same time, it was found that, on passing from the argument h to the argument $h/h_\infty = \eta$, one can obtain curves $\zeta(\eta)$ that almost coincide with one another in the interval of wind speeds from 7 to 20 m/sec.

Thus, for practical purposes it is quite permissible to confine oneself to a single universal dependence of ζ on η when calculating the elements of the field of wind waves for such wind speeds. Let us recall that, on the basis of ([5]), the transition from the first approximation to the second is effected by multiplying the abscissae of the diagrams by ζ and by $(h/h_\infty)^{1/2} = \eta^{1/2}$.

In this connection it is of interest to construct a curve representing the dependence of the full correction coefficient $\zeta\eta^{1/2}$ on the argument η. Such a curve is given in Fig. 2, where the values of η, taken from the first-approximation curve ([5]), are plotted along the abscissa axis, and the values of the full correction coefficient $\zeta\eta^{1/2}$ along the ordinate axis.

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Note: Figure translations are in progress. See original paper for figures.