DAMPING OF FREE OSCILLATIONS OF A GYROSCOPE IN A GIMBAL SUSPENSION WITH DRY FRICTION

`![Fig. 1](figure)`

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Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

Abstract

Full Text

MECHANICS

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DAMPING OF FREE OSCILLATIONS OF A GYROSCOPE IN A GIMBAL SUSPENSION WITH DRY FRICTION

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1. In the present note the motion of a balanced gyroscope in a gimbal suspension on a fixed base is considered; the gyroscope rotor and the rings are regarded as weightless, but, of course, as possessing mass. It is assumed that friction forces are present in the suspension axes and that their magnitude is proportional to the dynamic reactions; all other external actions are absent. An investigation carried out, as in work ([1]), with the aid of an "image" point showed that the free oscillations of the gyroscope are damped.

Fig. 1

Fig. 2

2. Let us associate with the fixed base, the outer ring, and the inner ring, respectively, three coordinate systems $\xi\eta\zeta$, $x_1y_1z_1$, $x_2y_2z_2$, with a common origin at the center of the gimbal suspension. Let us direct the axes ξ and x_1 along the axis of the outer ring, the axes y_1 and y_2 along the axis of the inner ring, and the axis z_2 along the axis of the gyroscope rotor. We shall determine the position of the gyroscopic system by the angles α, β, and φ, the positive directions of which are shown in Fig. 1.

Let, further, K_{x_1}, K_{y_1}, K_{z_1} be the sums of the moments of the forces of action of the base on the outer ring with respect to the axes x_1, y_1, z_1; L_{x_1}, L_{y_1}, L_{z_1} be the sums of the moments of the forces acting on the inner ring from the side of the outer ring; M_{x_2}, M_{y_2}, M_{z_2} be the moments of the action of the inner ring on the rotor ([2]).

Consider the interaction forces in one of the bearings of the outer ring. Following

I. I. Metelitsyn, we shall assume that a reaction force R_1 acts on the outer ring, making an angle ϑ_1 with the axis y_1 and perpendicular to the axis of rotation. The friction force $F_1 = f_1 R_1$ is directed perpendicular to the reaction force and impedes the rotation of the outer ring (Fig. 2). In the second bearing the magnitudes of the reaction and friction forces will be the same as in

first, and its direction is opposite. From the assumptions indicated it follows that

$$K_{x_1} = -2 f_1 r_1 R_1 \operatorname{sign} \alpha', \qquad K_{y_1} = -R_1 l_1 (\sin\vartheta_1 + f_1 \cos\vartheta_1 \operatorname{sign}\alpha'),$$

$$K_{z_1} = R_1 l_1 (\cos\vartheta_1 - f_1 \sin\vartheta_1 \operatorname{sign}\alpha'),$$

$$L_{x_1} = R_2 l_2 (\cos\vartheta_2 - f_2 \sin\vartheta_2 \operatorname{sign}\beta'), \qquad L_{y_1} = -2 f_2 r_2 R_2 \operatorname{sign}\beta',$$

$$L_{z_1} = -R_2 l_2 (\sin\vartheta_2 + f_2 \cos\vartheta_2 \operatorname{sign}\beta'),$$

$$M_{x_2} = -Rl(\sin\vartheta + f\cos\vartheta), \qquad M_{y_2} = Rl(\cos\vartheta - f\sin\vartheta), \qquad M_{z_2} = -2frR,$$

where $r_1,\ r_2,\ r$ are the radii of the bearings; $l_1,\ l_2,\ l$ are the lengths of the axes of the outer ring, the inner ring, and the rotor, respectively; ϑ_2 is the angle between the force R_2 exerted by the outer ring on the inner ring and the axis z_1; ϑ is the angle between the force R exerted by the inner ring on the rotor and the axis x_2.

Writing the equations of motion for the outer ring, the inner ring, and the rotor, we obtain a system of 9 differential equations

$$A_1 \alpha'' = -2 f_1 r_1 R_1 \operatorname{sign}\alpha' - R_2 l_2 (\cos\vartheta_2 - f_2 \sin\vartheta_2 \operatorname{sign}\beta'),$$

$$0 = -R_1 l_1 (\sin\vartheta_1 - f_1 \cos\vartheta_1 \operatorname{sign}\alpha') + 2 f_2 r_2 R_2 \operatorname{sign}\beta',$$

$$0 = R_1 l_1 (\cos\vartheta_1 - f_1 \sin\vartheta_1 \operatorname{sign}\alpha') + R_2 l_2 (\sin\vartheta_2 + f_2 \cos\vartheta_2 \operatorname{sign}\beta'),$$

$$A_2 \frac{d}{dt}(\alpha' \cos\beta) + (C_2 - B_2)\alpha'\beta' \sin\beta =$$

$$= R_2 l_2 (\cos\vartheta_2 - f_2 \sin\vartheta_2 \operatorname{sign}\beta') \cos\beta +$$

$$+R_2 l_2(\sin\vartheta_2 + f_2\cos\vartheta_2 \operatorname{sign}\beta')\sin\beta + Rl(\sin\vartheta + f\cos\vartheta), \qquad (1)$$

$$B_2\beta'' + (A_2 - C_2)\alpha a'^2 \sin\beta\cos\beta = -2f_2 r_2 R_2 \operatorname{sign}\beta' - Rl(\cos\vartheta - f\sin\vartheta),$$

$$C_2\frac{d}{dt}(\alpha'\sin\beta) + (B_2 - A_2)\alpha'\beta'\cos\beta =$$

$$= R_2 l_2(\cos\vartheta_2 - f_2\sin\vartheta_2 \operatorname{sign}\beta')\sin\beta -$$

$$-R_2 l_2(\sin\vartheta_2 + f_2\cos\vartheta_2 \operatorname{sign}\beta')\cos\beta + 2frR,$$

$$A\frac{d}{dt}(\alpha'\cos\beta) + H\beta' - A\alpha'\beta'\sin\beta = -Rl(\sin\vartheta + f\cos\vartheta),$$

$$A\beta'' + A\alpha a'^2\sin\beta\cos\beta - H\alpha'\cos\beta = Rl(\cos\vartheta - f\sin\vartheta),$$

$$\frac{d}{dt}\left[C(\alpha'\sin\beta + \varphi')\right] = \frac{dH}{dt} = -2frR.$$

3. Suppose that there is no friction in the rotor axis, i.e. $f = 0$. Considering the angles α, β and the angular velocities α', β' to be small quantities and neglecting their squares, products, and the term $\alpha''\beta$, we obtain from (1) the system of equations

$$J_1 x' + Hy = -a_1|J_3 y' - Hx| \operatorname{sign} x,$$

$$J_3 y' - Hx = -a_2|J_2 x' + Hy| \operatorname{sign} y. \qquad (2)$$

Here

$$\alpha' = x, \qquad \beta' = y, \qquad A + A_1 + A_2 = J_1, \qquad A + A_2 = J_2, \qquad A + B_2 = J_3,$$

$$\frac{2f_1 r_1}{l_1\sqrt{1+f_1^2}} = a_1 > 0, \qquad \frac{2f_2 r_2}{l_2\sqrt{1+f_1^2}} = a_2 > 0.$$

4. Let us consider the motion of the "representing" point in the plane x, y. It can be shown that:

1) in the first quadrant, where $x > 0,\ y > 0$, and in the third quadrant, where $x < 0,\ y < 0$, the motion of the gyroscope is described by the equations

$$J_1 x' + Hy = a_1(J_3 y' - Hx), \qquad x' = -\frac{1 + a_1 a_2}{J_1 + a_1 a_2 J_2} Hy; \tag{3}$$

2) in the second quadrant, where $x < 0,\ y > 0$, and in the fourth quadrant, where $x > 0,\ y < 0$, it takes the form

$$J_1 x' + Hy = -a_1(J_3 y' - Hx), \qquad x' = -\frac{1 - a_1 a_2}{J_1 - a_1 a_2 J_2} Hy. \tag{4}$$

Denoting

$$m_1 = \frac{J_1 + a_1 a_2 J_2}{J_3(1 + a_1 a_2)}, \qquad m_2 = \frac{J_1 - a_1 a_2 J_2}{J_3(1 - a_1 a_2)}, \qquad n_1 = \frac{a_2 A_1}{J_3(1 + a_1 a_2)}, \qquad n_2 = \frac{a_2 A_1}{J_3(1 - a_1 a_2)}.$$

we write (3) and (4) in the form of a single equation

$$\frac{dy}{dx} = -m_i \frac{x}{y} + n_i \qquad (i = 1, 2), \tag{5}$$

integrating which, we obtain

$$\ln(y^2 - n_{ixy} + m_{ix}^2) = M - \frac{n_i}{p_i} \operatorname{arc\,tg} \frac{2y - n_{ix}}{2p_{ix}} \qquad (i = 1, 2), \tag{6}$$

where $p_i^2 = m_i - n_i^2/4$.

From relation (6) we find

$$y_k = y_0 \left(\frac{m_1}{m_2}\right)^k \exp\left\{k\left[\frac{n_2}{p_2} \operatorname{arc\,tg} \frac{n_2}{2p_2} - \frac{n_1}{p_1} \operatorname{arc\,tg} \frac{n_1}{2p_1} - \left(\frac{n_1}{p_1} + \frac{n_2}{p_2}\right)\frac{\pi}{2}\right]\right\}; \tag{7}$$

$$x_k = x_0 \left(\frac{m_1}{m_2}\right)^k \exp\left\{k\left[\frac{n_2}{p_2} \operatorname{arc\,tg} \frac{n_2}{2p_2} - \frac{n_1}{p_1} \operatorname{arc\,tg} \frac{n_1}{2p_1} - \left(\frac{n_1}{p_1} + \frac{n_2}{p_2}\right)\frac{\pi}{2}\right]\right\}, \tag{8}$$

where $x_0,\ y_0$ are the magnitudes of the segments cut off on the coordinate axes by the point $P(x, y)$ at the beginning of the motion, and $x_k,\ y_k$ after k of its revolutions around the origin. From formulas (7) and (8) it is clear that the natural oscillations of the gyroscope decay with time, since $m_2 > m_1$, while a_1 and a_2 are usually much smaller than 1, and therefore $\operatorname{arc\,tg} \frac{n_2}{2p_2} - \frac{\pi}{2} < 0$.

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CITED LITERATURE

[1] E. L. Nikolai, *Works on Mechanics*, Moscow, 1955.
[2] A. Yu. Ishlinskii, *Applied Mathematics and Mechanics*, 21, issue 1 (1957).

Note: Figure translations are in progress. See original paper for figures.