HYDRAULICS

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1958

SovietRxiv

Abstract

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ON THE QUESTION OF THE EXTINCTION OF TRANSVERSE CIRCULATION

(Presented by Academician L. I. Sedov on 22 X 1957)

Natural transverse circulation, arising in curvilinear sections of channels, gradually dies out along the length of a straight channel. Beyond a bend, at some distance, the transverse velocities become so small that in practice the flow may be regarded as parallel-stream. The present work is devoted to finding the law of extinction in sections where the factors that caused the circulation are absent, while the circulation in the initial section, immediately after the bend, is regarded as given.

The starting point is the equations of motion of a viscous incompressible fluid and the equation of continuity, with the coefficient of turbulent mixing assumed constant:

$$\begin{gathered} v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{A_0}{\rho}\Delta^2 v_x, \\ v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{A_0}{\rho}\Delta^2 v_y, \\ v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} = g - \frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{A_0}{\rho}\Delta^2 v_z, \\ \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0. \end{gathered} \tag{1}$$

The origin of the coordinate system lies on the free surface of the flow; the axis x is directed along the flow; y is the horizontal transverse axis; z is the vertical axis directed downward; v_x, v_y, v_z are the velocities along the corresponding axes.

It is assumed that the longitudinal velocities remain constant, and that the pressure obeys the hydrostatic law. With the aid of a stream function, through which the transverse velocities are expressed as $v_y = +\partial F/\partial z$ and $v_z = -\partial F/\partial y$, the equation of continuity and the first equation of system (1) are simultaneously satisfied.

Since the transverse velocities are small in comparison with the longitudinal velocities, and their variations along the axes are of the same order, in the first approximation the nonlinear terms in system (1) may be neglected, and for determining v_y and v_z we obtain

$$\frac{\partial^2 v_y}{\partial \xi^2} + \frac{1}{a^2}\frac{\partial^2 v_y}{\partial \eta^2} + \frac{\partial^2 v_y}{\partial \zeta^2} - b\frac{\partial v_y}{\partial \xi} = 0; \tag{2}$$

$$\frac{\partial^2 v_z}{\partial \xi^2} + \frac{1}{a^2}\frac{\partial^2 v_z}{\partial \eta^2} + \frac{\partial^2 v_z}{\partial \zeta^2} - b\frac{\partial v_z}{\partial \xi} = 0. \tag{3}$$

Here the following relative coordinates and notation have been introduced:

$$\xi = \frac{x}{H}, \qquad \eta = \frac{y}{B}, \qquad \zeta = \frac{z}{H}, \qquad a = \frac{B}{H}, \qquad b = \frac{v_0 \rho H}{A_0}, \tag{4}$$

where B is the width of the channel and H is the depth.

The solution of equations (2) and (3) is carried out by the method of separation of variables for linear second-order partial differential equations. As a result we obtain

$$v_y = \sum_i \sum_j \left\{ A_{ij} \exp\left[\left(\frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 + k_i^2}\right)\xi\right] + \right.$$
$$\left. + B_{ij} \exp\left[-\left(-\frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 + k_i^2}\right)\xi\right]\right\} \times$$
$$\times \left(C_{ij} \cos a\sqrt{k_i^2 - n_j^2}\,\eta + \sin a\sqrt{k_i^2 - n_j^2}\,\eta\right)\left(\cos n_j\zeta + F_j \sin n_j\zeta\right); \tag{5}$$

$$v_z = \sum_i \sum_j \left\{ L_{ij} \exp\left[\left(\frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 + k_i^2}\right)\xi\right] + \right.$$
$$\left. + M_{ij} \exp\left[-\left(-\frac{b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 + k_i^2}\right)\xi\right]\right\} \times$$
$$\times \left(\cos a\sqrt{k_i^2 - n_j^2}\,\eta + N_{ij} \sin a\sqrt{k_i^2 - n_j^2}\,\eta\right)\left(K_j \cos n_j\zeta + \sin n_j\zeta\right). \tag{6}$$

The boundary conditions for determining the arbitrary constants entering into (5) are: a) impermeability of the side walls and b) equality to zero of the tangential stresses on the bottom and on the free surface of the flow. In this case we obtain

$$C_{ij} = 0, \qquad k_i = \sqrt{\frac{m^2\pi^2}{a^2} + n_j^2}\ (m = 1, 2, ...), \qquad F_j = 0, \tag{7}$$

$$n_j = n\pi\ (n = 1, 2, ...).$$

The conditions for the vertical component of velocity are the following: a) impermeability of the bottom and of the free surface of the flow, b) equality to zero of the tangential stresses on the side walls. The latter conditions make it possible to determine the parameters

$$N_{ij} = 0, \qquad n_j = n\pi\ (n = 1, 2, ...),$$

$$K_j = 0, \qquad k_i = \sqrt{\frac{m^2\pi^2}{a^2} + n^2\pi^2}\ (m = 1, 2, ...). \tag{8}$$

From the condition of absence of circulation in the infinitely distant section of the rectilinear reach it follows that, for all values of i and j,

$$A_{ij} = L_{ij} = 0. \tag{9}$$

The values of the parameters (7)–(9) make it possible to determine uniquely the transverse velocities from equations (5) and (6):

$$v_y = \sum_m \sum_n B_{mn} \exp\left(-\delta_{mn}\frac{x}{H}\right) \sin\frac{m\pi y}{B} \cos\frac{n\pi z}{H}; \tag{10}$$

$$v_z = \sum_m \sum_n M_{mn} \exp\left(-\delta_{mn}\frac{x}{H}\right) \cos\frac{m\pi y}{B} \sin\frac{n\pi z}{H}, \tag{11}$$

where

$$\delta_{mn} = -\frac{v_0\rho H}{2A_0} + \sqrt{\left(\frac{v_0\rho H}{2A_0}\right)^2 + \pi^2\left(\frac{H^2}{B^2}m^2 + n^2\right)}. \tag{12}$$

For the purpose of a qualitative analysis of the solution obtained, we express the coefficient of turbulent mixing by the empirical formula of Karaushe–

(1), after which the decrement of attenuation takes the form

$$\delta_{mn} = -\frac{MC}{2g} + \sqrt{\left(\frac{MC}{2g}\right)^2 + \pi^2\left[\left(\frac{H}{B}\right)^2 m^2 + n^2\right]}, \tag{13}$$

Fig. 1. Attenuation of transverse circulation in channels with different width and roughness

Figure 1: Fig. 1. Attenuation of transverse circulation in channels with different width and roughness

Fig. 2. Drop in the energy of transverse circulation

Figure 2: Fig. 2. Drop in the energy of transverse circulation

where C is the Chézy coefficient; M is a coefficient which, according to Bazin, varies from 40 to 48, according to Boussinesq is equal to 44.6, and according to Karaushev varies depending on the Chézy coefficient. V. M. Makkaveev ([2]) recommends taking M according to Boussinesq.

Fig. 1. Attenuation of transverse circulation in channels with different width and roughness

The coefficients B_{mn} and M_{mn} are determined from the conditions for a prescribed distribution of transverse velocities in the initial cross section of the straight reach. However, taking into account the rapid attenuation of terms with large ordinal numbers, one may restrict oneself to several terms of the series (10) and (11). In particular, the attenuation of the root-mean-square velocity of circulation, which more accurately characterizes the intensity of circulation, has the form

$$\overline{v}_c = \overline{v}_0 \exp\left(-\delta_{11}\frac{x}{H}\right), \tag{14}$$

where δ_{11} is obtained from (13) for $m = n = 1$, and $\overline{v}_0 = \overline{v}_c$ (for $x = 0$) is regarded as given.

The expressions obtained show that the law of attenuation of circulation depends on the geometric dimensions of the channel and on its roughness.

In Fig. 1 are given curves of attenuation of the root-mean-square velocity of circulation for different ratios of channel width to depth and for different values of the Chézy coefficient. Analysis of these curves shows that, for all ratios B/H, circulation attenuates more intensively in rough channels. For one and the same roughness, circulation attenuates more intensively in narrow channels.

Fig. 2. Drop in the energy of transverse circulation:
1 –theoretical curve; 2, 3, and 4 –Bolshakov' s experimental curves: 2 –for $v_{\text{cp}} = 20$ cm/sec, 3 –for $v_{\text{cp}} = 30$ cm/sec, 4 –for $v_{\text{cp}} = 40$ cm/sec

The author carried out a comparison of transverse velocities (10) and (11) and of the root-mean-square velocity of circulation (14) with the experimental data of M. V. Potapov ([3]), M. Yu. Vagabov ([4]), and A. A. Bolshakov ([5]), and with data from the hydroelectric laboratory of the Water-Energy Institute of the Academy of Sciences of the Armenian SSR.

A. A. Bolshakov, on the basis of experiments in an experimental flume, on the pro-

with a length of 160 cm—at intervals of every 20 cm—gives the value of the energy of the transverse circulation $h_c = \bar{v}_c^2/2g$ (flume width 40 cm, filling depth 12 cm, and Chézy coefficient $C = 40$). In Fig. 2, for illustration, a curve is given for the decrease of the energy of the transverse circulation

$$h_c = h_0 \exp\left(-2\delta_{11}\frac{x}{H}\right)$$

and Bolshakov's data for different values of the mean velocity. Along the abscissa is plotted the length of the canal, and along the ordinate—the circulation energy in dimensionless quantities. The deviation was always within permissible limits.

The work was carried out under the supervision of Prof. A. K. Ananyan, to whom the author expresses his gratitude.

Received
14 IX 1957

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Note: Figure translations are in progress. See original paper for figures.