

# ON A HYDRODYNAMIC FORECAST OF THE BARIC AND KINEMATIC FIELDS (SHORT-RANGE WEATHER FORECAST)

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**Abstract**

**Full Text**

**GEOPHYSICS**

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**ON A HYDRODYNAMIC FORECAST OF THE  
BARIC AND KINEMATIC FIELDS (SHORT-  
RANGE WEATHER FORECAST)**

*(Presented by Academician V. V. Shuleikin, 27 V 1958)*

We shall proceed from the basic system of equations of hydromechanics and thermodynamics of an ideal fluid under the assumption of an adiabatic process.

Let  $T, p$  denote the temperature and pressure of atmospheric air;  $u, v, w$  the velocity components;  $g$  the acceleration due to gravity;  $l = 2\omega \sin \varphi$  the Coriolis parameter, where  $\omega$  is the angular velocity of the earth's rotation and  $\varphi$  is the latitude of the place. In the local problem we shall assume  $l = \text{const}$ . In addition, let  $T' = T - T_0$ , where  $T_0$  is a constant quantity (for example,  $273^\circ$ );

$$\vartheta = \frac{T'}{T_0};$$

$$Q = RT_0 \ln \frac{p}{p_0} + gz; \quad c^2 = \frac{c_p}{c_v} RT_0,$$

where  $p_0 = 1000$  mb and  $R$  is the gas constant;  $A$  is the thermal equivalent of work.

We write the system of equations of our problem in the form

$$\begin{aligned} u_t - lv + Q_x &= -(\vartheta Q_x + uu_x + vv_y + ww_z) \equiv F_1, \\ v_t + lu + Q_y &= -(\vartheta Q_y + uv_x + vv_y + wv_z) \equiv F_2, \\ w_t + Q_z &= -[\vartheta(Q_z - g) + uw_x + vw_y + ww_z] \equiv F_3, \\ \frac{1}{c^2} Q_z + u_x + v_y + w_z &= -\frac{1}{c^2} [uQ_x + vQ_y + w(Q_z - g)] \equiv F_4; \end{aligned} \tag{1}$$

$$\vartheta_t + \frac{AR}{c_v} (u_x + v_y + w_z) = \frac{AR}{c_v} \vartheta (u_x + v_y + w_z) - (w\vartheta_x + v\vartheta_y + w\vartheta_z) \equiv F_5. \tag{2}$$

Let us denote the initial values at  $t = 0$  by

$$u = \overset{0}{u}, \quad v = \overset{0}{v}, \quad w = \overset{0}{w}, \quad Q = \overset{0}{Q}, \quad \vartheta = \overset{0}{\vartheta}.$$

System (1) can be reduced to an integro-differential one by solving it with respect to  $u, v, w, Q$ , expressing them through  $F_i$ . Denote these solutions, satisfying the initial conditions, for the homogeneous system of differential equations by  $u_1, v_1, w_1, Q_1$ , and for the inhomogeneous system (1) with zero initial values by  $u_2, v_2, w_2, Q_2$

$$(u = u_1 + u_2, \quad v = v_1 + v_2, \quad w = w_1 + w_2, \quad Q = Q_1 + Q_2).$$

Both the homogeneous and the inhomogeneous systems are solved by the operational method using the Laplace-Carson and Fourier transforms analogously to (2).

For the homogeneous system (1), at  $z = 0$  the condition  $w = 0$  gives  $Q_{1z} = 0$ , and, for example, for  $Q_1$  then

$$Q_1 = -\frac{1}{4\pi} \int_{-\infty}^{+\infty} \int_0^{\infty} \int_{-\infty}^{\infty} \left( \sum_{k=1}^4 \Phi_k^{(Q)} G_{3-k}^{(+)} \right) dz' dx' dy', \quad (3)$$

where

$$G_m^{(\pm)} = \frac{1}{R_1} \frac{\partial^m}{\partial t^m} J_0 \left( \frac{lr}{R_1} \sqrt{t^2 + \left( \frac{R_1}{c} \right)^2} \right) \pm \frac{1}{R_2} \frac{\partial^m}{\partial t^m} J_0 \left( \frac{lr}{R_2} \sqrt{t^2 - \left( \frac{R_2}{c} \right)^2} \right),$$

$$G_{-1}^{(\pm)} = \frac{1}{R_1} \int_{R_1/c}^t J_0 \left( \frac{lr}{R_1} \sqrt{\tau^2 - \left( \frac{R_1}{c} \right)^2} \right) d\tau \pm \frac{1}{R_2} \int_{R_2/c}^t J_0 \left( \frac{lr}{R_2} \sqrt{t^2 - \left( \frac{R_2}{c} \right)^2} \right); \quad (4)$$

$$R_1^2 = (x' - x)^2 + (y' - y)^2 + (z' - z), \quad R_2^2 = (x' - x)^2 + (y' - y)^2 + (z' + z)^2,$$

$$r^2 = (x' - x)^2 + (y' - y)^2; \quad (5)$$

$$\Phi_1^{(Q)} = -\frac{\overset{0}{Q}}{c^2}, \quad \Phi_2^{(Q)} = \overset{0}{u}_x + \overset{0}{v}_y + \overset{0}{w}_z,$$

$$\Phi_3^{(Q)} = -\frac{l^2 \overset{0}{Q}}{c^2} + l(\overset{0}{v}_x - \overset{0}{u}_y), \quad \Phi_4^{(Q)} = l^2 \overset{0}{w}_z. \quad (6)$$

(3) shows that, in order to compute the pressure field  $Q_1$ , it is not enough to have initial pressure data for one time; it is also necessary to have the components of the initial velocities. However, one can obtain a solution that would depend only on the initial  $Q$  and its time derivatives.

Indeed, if from the homogeneous system (1) all unknowns except  $Q$  are eliminated, then for  $Q_1$  we obtain

$$\left[ \frac{\partial^2}{\partial t^2} \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta \right) + l^2 \left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2} \right) \right] Q_1 = 0, \quad \text{where } \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}. \quad (7)$$

The solution of (7) is carried out by the method used earlier. We obtain

$$\begin{aligned} Q_1 = & \frac{1}{4\pi} \int_{-\infty}^{\infty} \iint \left[ \frac{Q}{c^2} \frac{1}{R_1} \frac{\partial^2}{\partial t^2} J_0 \left( \frac{lr}{R_1} \sqrt{t^2 - \left( \frac{R_1}{c} \right)^2} \right) + \right. \\ & + \frac{Q_t}{c^2} \frac{1}{R_1} \frac{\partial}{\partial t} J_0 \left( \frac{lr}{R_1} \sqrt{t^2 - \left( \frac{R_1}{c} \right)^2} \right) + \left( \frac{Q_{tt}}{c^2} - \Delta Q \right) \frac{1}{R_1} J_0 \left( \frac{lr}{R_1} \sqrt{t^2 - \left( \frac{R_1}{c} \right)^2} \right) + \\ & \left. + \left( \frac{Q_{ttt}}{c^2} - \Delta Q_t \right) \frac{1}{R_1} \int_{R_1/c}^t J_0 \left( \frac{lr}{R_1} \sqrt{t^2 - \left( \frac{R_1}{c} \right)^2} \right) d\tau \right] dx' dy' dz', \quad (8) \end{aligned}$$

where one should take  $Q(x, y, -z) = Q(x, y, z)$ .

According to (8), to predict the pressure field it is necessary to have data on the pressure and its three time derivatives; in other words, to have pressure data for four times. Solution (8) indicates the propagation of damped oscillations with the speed of sound, which is in agreement with (5).

If it is assumed that  $\frac{1}{c^2} \approx 0$  (which is equivalent to  $\partial\rho/\partial t \approx 0$ ), then (8) can be written as follows:

$$Q_1 = Q + \frac{1}{4\pi} \int_{-\infty}^{+\infty} \iint \frac{\Delta Q}{R_1} \left[ 1 - J_0 \left( \frac{lr t}{R_1} \right) \right] dx' dy' dz' - \frac{1}{4\pi} \int_{-\infty}^{\infty} \iint \frac{\Delta Q_t}{R_1} \int_0^t J_0 \left( \frac{lr \tau}{R_1} \right) d\tau dx' dy' dz'. \quad (9)$$

In this case the history of the development of synoptic processes is sufficiently illuminated by data on the pressure field and its time derivative, or, in other words, by pressure data for two times. Solution (9) (for  $\partial\rho/\partial t = 0$ ) was obtained by another method by S. L. Sobolev <sup>(6)</sup>.

The nonhomogeneous system of equations (1) is solved in a completely analogous way. In particular, we find for  $Q_2$

$$\begin{aligned}
 Q_2 = & -\frac{1}{4\pi} \iiint_{-\infty}^{\infty} \int^{t-R_1/c} \left\{ F_4 \frac{1}{R_1} \frac{\partial^2}{\partial t^2} J_0 \left( \frac{lr}{R_1} \sqrt{(t-\tau)^2 - \left(\frac{R_1}{c}\right)^2} \right) - \right. \\
 & - (F_{1x} + F_{2y} + F_{3z}) \left( \frac{1}{R_1} \frac{\partial}{\partial t} J_0 \left( \frac{lr}{R_1} \sqrt{(t-\tau)^2 - \left(\frac{R_1}{c}\right)^2} \right) \right) + \\
 & + [l^2 F_4 + l(F_{1y} - F_{2x})] \frac{1}{R_1} J_0 \left( \frac{lr}{R_1} \sqrt{(t-\tau)^2 - \left(\frac{R_1}{c}\right)^2} \right) - \\
 & \left. - l^2 F_{3z} \int_{R_1/c}^{t-\tau} J_0 \left( \frac{lr}{R_1} \sqrt{\tau_1^2 - \left(\frac{R_1}{c}\right)^2} \right) d\tau_1 \right\} d\tau dx' dy' dz'.
 \end{aligned} \tag{10}$$

Under the conditions  $\frac{1}{c^2} \approx 0$ ,  $\vartheta \approx 0$ , in the geostrophic approximation, when

$$u = \frac{1}{l} Q_y, \quad v = -\frac{1}{l} Q_x,$$

we obtain:

$$\begin{aligned}
 Q_2 \approx & \frac{1}{2\pi l^2} \int_{-\infty}^{\infty} \iiint_0^t (Q_x, Q_y) \frac{1}{R_1} \frac{\partial}{\partial t} J_0 \left( \frac{lr(t-\tau)}{R_1} \right) d\tau dx' dy' dz' + \\
 & + \frac{1}{2\pi l} \int_{-\infty}^{\infty} \iiint_0^t (\Delta' Q, Q) \frac{1}{R_1} J_0 \left( \frac{lr(t-\tau')}{R_1} \right) d\tau dx' dy' dz'.
 \end{aligned} \tag{11}$$

Here

$$\Delta' = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad \text{and} \quad (A, B) = \frac{D(A, B)}{D(x, y)}.$$

On the right-hand side of (11), the functions  $Q$  enter for the variable  $t$ . Therefore the equality  $Q = Q_1 + Q_2$ , upon insertion of (11) into it, is an integro-differential equation with respect to  $Q$ . Its solution can be carried out numerically “step by step,” taking  $Q = \overset{0}{Q}$  at each step (a similar path was proposed by I. A. Kibel for another system (4)).

The solution of the nonlinear system (1), (2) can, under known conditions, also be performed without reduction to an integro-differential one; for this purpose it

is reduced to a sequence of recurrent systems <sup>(1,3,7)</sup>. The first system coincides with the homogeneous (1); the second with the nonhomogeneous (1) is obtained by replacing the functions on the right-hand sides by their initial values.

The solution (10) for the half-space is easily obtained, as for  $Q_1$ , under the assumption  $Q_z|_{z=0} \approx 0$ . Precisely, from  $w|_{z=0} = 0$  it follows from (1) that

$$Q_z = g \frac{\vartheta}{1 + \vartheta}. \quad (12)$$

Then, in reducing to the integro-differential system, in (12)  $\vartheta$  at  $z = 0$  should be regarded as a known function of  $x, y, t$ , and in the solution (10) only an extra term will appear. The solution is then carried out "step by step." The changes in  $\vartheta$  are determined from (2).

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*Note: Figure translations are in progress. See original paper for figures.*

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