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Abstract

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PHYSICS

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ON A VARIATIONAL PRINCIPLE IN THE MANY-BODY PROBLEM

Consider a dynamical system of Fermi particles with a Hamiltonian of the form

$$H = \sum \{T(f, f') - \lambda \delta_{f,f'}\} a_f^+ a_{f'} + \frac{1}{2} \sum J(f_1, f_2, f_2', f_1') a_{f_1}^+ a_{f_2}^+ a_{f_2'} a_{f_1'}, \qquad (1)$$

where λ is the chemical potential; $a,\ a^+$ are Fermi amplitudes; f is the set of indices characterizing the state of one particle.

We perform a linear transformation of the Fermi amplitudes:

$$a_f = \sum_\nu (u_{f\nu} \alpha_\nu + v_{f\nu} \alpha_\nu^+). \qquad (2)$$

In order that this transformation be canonical and thereby not violate the commutation properties of the Fermi amplitudes, the c-functions $u,\ v$ must satisfy the orthonormality conditions

$$\xi_{f,f'} \equiv \sum_\nu \{u_{f\nu} u_{f'\nu}^* + v_{f\nu} v_{f'\nu}^*\} = \delta_{f,f'},$$

$$\eta_{f,f'} \equiv \sum_\nu \{u_{f\nu} v_{f'\nu} + v_{f\nu} u_{f'\nu}\} = 0. \qquad (3)$$

Substitute (2) into expression (1) and find the mean value of H with respect to the vacuum state C_0:

$$\alpha_\nu C_0 = 0$$

for the new Fermi amplitudes. We obtain

$$\overline{H} = \sum \{T(f,f') - \lambda\delta_{f,f'}\} F_1(f,f') +$$

$$+\frac{1}{2}\sum J(f_1,f_2,f_2',f_1')\{\Phi^*(f_1,f_2)\Phi(f_1',f_2') +$$

$$+F_1(f_1,f_1')F_1(f_2,f_2') - F_1(f_2,f_1')F_1(f_1,f_2')\} \equiv \mathcal{E}(u,v), \tag{4}$$

where

$$F_1(f,f') = \sum_{(\nu)} v^*_{f\nu} v_{f'\nu}, \qquad \Phi(f,f') = \sum_{(\nu)} v_{f\nu} u_{f'\nu}.$$

We determine u, v from the condition of a minimum of the form $\mathcal{E}(u,v)$ in the presence of the additional conditions (3). The corresponding stationarity equation will be

$$\delta\tilde{\mathcal{E}}(u,v) = 0, \tag{5}$$

$$\tilde{\mathcal{E}}(u,v) = \mathcal{E}(u,v) + \sum_{f,f'} \{\lambda(f,f')\xi(f,f') + \mu(f,f')\eta(f,f') + \mu^*(f,f')\eta^*(f,f')\};$$

where λ, μ are Euler multipliers; the variations $\delta u, \delta v$ and $\delta u^*, \delta v^*$ are regarded here as independent.

We now arrive at the formulation of a new approximate method in the many-body problem. In this method we take such u, v, satisfying the stationarity equations, that give the minimal value to the form $\mathcal{E}(u,v)$. For them the corresponding C_0 is regarded as the wave function of the ground state, and $\mathcal{E}(u,v)$ as the energy of the ground state. The question of the justification of the method and of the limits of applicability is rather complicated. Therefore here we shall confine ourselves only to a number of remarks.

Thus, on the basis of the results of work ([1]), we may assert that the proposed method gives an exact solution of the problem in the case when in the Hamiltonian only interactions of pairs of particles with opposite momenta are taken into account. On the other hand, we shall show that among the solutions of the stationarity equation there is always a solution corresponding exactly to the well-known Fock method ([2]).

Indeed, take a system of functions $\varphi_{f\nu}$, orthonormalized in the usual sense,

$$\zeta(f,f') \equiv \sum_{\nu} \varphi^*_{f\nu}\varphi_{f'\nu} = \delta_{f,f'} \tag{6}$$

and divide the whole set of indices ν into two parts F and G. As F—the "Fermi sphere" —we take a finite set of indices ν, consisting of N elements (where N is the number of particles); the remaining ν we combine into an additional set G.

Set

$$\begin{aligned} u_{f\nu} &= 0, & v_{f\nu} &= \varphi_{f\nu}, & \nu &\in F; \\ u_{f\nu} &= \varphi_{f\nu}, & v_{f\nu} &= 0, & \nu &\in G. \end{aligned} \tag{7}$$

Then, obviously, all the orthogonality conditions (3) will be satisfied. If such u, v are substituted into the form $\mathcal{E}$, then Φ vanishes in it, and it will depend only on F_1, and thereby only on $\varphi_{f\nu}$ for $\nu \in F$.

We agree to denote $\nu \in F$ by the letter ω. Determine $\varphi_{f\omega}$ from the condition of the minimum of the form $\mathcal{E}(\cdots\varphi_{f\omega}\cdots)$ under the additional conditions (6). The corresponding stationarity equation will be

$$\delta\tilde{\mathcal{E}}_F = 0, \qquad \tilde{\mathcal{E}}_F = \mathcal{E}(\cdots\varphi_{f\omega}\cdots) + \sum_{f,f'} \lambda(f, f')\zeta(f, f'). \tag{8}$$

It is not difficult to notice that we have now formulated nothing other than the usual Fock method. The wave function of the system corresponds to such a position when the individual particles occupy all the states $\varphi_{f\omega}$; the remaining states $\varphi_{f\nu}$ are empty. On the other hand, from equations (5) we see that they always have a solution of type (7), in which the $\varphi_{f\omega}$ are chosen by the Fock method as solutions of equations (8). Thus, our method may be regarded as a generalization of the Fock method and, consequently, its limits of applicability, in any case, will not be narrower.

Arguing as in work (^{1}) and composing the expression for the second variation $\delta^2\mathcal{E}(u, v)$ for the "normal solution" (7), we can obtain the condition for its instability. This condition is formulated with the aid of the eigenvalue problem for the corresponding system of linear equations. In practice it can be used, for example, to obtain a criterion of superconductivity in a model in which the crystal lattice of the metal is explicitly taken into account.

In conclusion, let us note that the method set forth can receive further development and refinement by means of studying the chain of equations for "distribution functions" :

$$\overline{\alpha^+_{f_1} \dots \alpha^+_{f_s} \alpha_{f'_r} \dots \alpha_{f'_1}} = F_{s+2}(t, f_1, \dots, f_s; f'_r, \dots, f'_1).$$

Thus, for example, if one takes the stationary case and retains in the equations of the chain only the functions $F_{0+2}(f_1, f_2)$, $F_{2+0}(f'_1, f'_2)$, while neglecting the others, then we again obtain the equations of our method. Taking the case when F_{0+2}, F_{2+0} explicitly depend on time, and restricting ourselves to a linear approximation in the deviations $F_{0+2} - F_{0+2}^{\mathrm{st}}$, $F_{2+0} - F_{2+0}^{\mathrm{st}}$, we obtain equations for determining the spectrum of collective oscillations.

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CITED LITERATURE

[1] N. N. Bogolyubov, DAN, **119**, No. 1 (1958). [2] V. A. Fock, *Zs. f. Phys.*, **61**, 126 (1930).

Note: Figure translations are in progress. See original paper for figures.

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