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Abstract

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MATHEMATICS

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NOMOGRAPHIC METHODS FOR THE APPROXIMATE REPRESENTATION OF A FUNCTION OF ONE VARIABLE

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Equations

$$\Phi[v, f(u)\cos\alpha - g(u)\sin\alpha + A, f(u)\sin\alpha + g(u)\cos\alpha + B] = 0; \qquad (1)$$

$$F[f(u,v)\cos\alpha + g(u,v)\sin\alpha + A, -f(u,v)\sin\alpha + g(u,v)\cos\alpha + B] = 0, \quad (2)$$

relating the variables u and v with three parameters A, B, and α, can be represented by transparent nomograms with three degrees of freedom of displacement of the transparent overlay[5].

On the fixed plane of the nomogram for relation (1) there is a family of lines v, whose equation is $\Phi(v, x, y) = 0$; on the transparent overlay there is a scale u, constructed according to the equations $x = f(u)$, $y = g(u)$. On the fixed plane of the nomogram for relation (2) there is a field (u, v), whose equations are $x = f(u, v)$, $y = g(u, v)$; on the transparent overlay there is a mute scale L, specified by the equation $F(x, y) = 0$. In addition, both nomograms have elements corresponding to the parameters A, B, and α. To each position of the transparent overlay on the fixed plane of the nomogram of equation (1) or (2) there correspond certain values of the parameters A, B, and α.

Equations (1) and (2) and their special cases can be used for solving, by nomographic methods, the following problems connected with the approximate representation of functions of one variable:

Problem 1. Interpolation by means of formulas (1) and (2).

Problem 2. Selection of parameters of empirical formulas of types (1) and (2).

Problem 3. Investigation of the absolute and relative errors when replacing a given function $\varphi(u)$ by a function from a three-parameter family defined respectively by the equations

$$F\{f[u,\varphi(u)-\Delta]\cos\alpha+g[u,\varphi(u)-\Delta]\sin\alpha+A,$$

$$-f[u,\varphi(u)-\Delta]\sin\alpha+g[u,\varphi(u)-\Delta]\cos\alpha+B\}=0, \tag{3}$$

$$F\{f[u,(1-\delta)\varphi(u)]\cos\alpha+g[u,(1-\delta)\varphi(u)]\sin\alpha+A,$$

$$-f[u,(1-\delta)\varphi(u)]\sin\alpha+g[u,(1-\delta)\varphi(u)]\cos\alpha+B\}=0, \tag{4}$$

belonging to type (2). Here Δ is the absolute error, and δ the relative error.

Problem 4. Investigation of the possibility of approximate representation of a given function $\varphi(u)$ by a function from a four-parameter family defined by the equation

$$F\{f[u,\varphi(u),C]\cos\alpha+g[u,\varphi(u),C]\sin\alpha+A,$$

$$-f[u,\varphi(u),C]\sin\alpha+g[u,\varphi(u),C]\cos\alpha+B\}=0, \tag{5}$$

belonging to type (2).

Problem 1 (determination of v from the given values $u, u_1, v_1, u_2, v_2, u_3, v_3$) is solved on the nomograms of equations (1) or (2) by a single placement of the tracing sheet.

To solve problem 2, one finds such a position of the tracing sheet on the fixed plane of the nomogram of equation (1) or (2) in which the contacts $u_1 rightarrow v_1$, $u_2 rightarrow v_2$, $u_3 rightarrow v_3, \dots, \dots, u_n rightarrow v_n$ are best satisfied (by eye) in the case of the nomogram of equation (1), and the contacts $L rightarrow (u_1, v_1)$, $L rightarrow (u_2, v_2)$, $L rightarrow (u_3, v_3), \dots, L rightarrow (u_n, v_n)$ in the case of the nomogram of equation (2), where u_1 and v_1, u_2 and v_2, etc., are experimental values of u and v.

The solution of problem 3 by means of the nomogram of equation (3) or (4) is based on the fact that, by moving the tracing sheet over the fixed plane of the nomogram, one can obtain a visual representation of the influence of the parameters A, B, and α on the character of the variation of the quantities Δ or δ as functions of u. This makes it possible easily to choose an approximation satisfying one or another requirement, for example in the Chebyshev sense.

The solution of problem 4 by means of the nomogram of equation (5) is based on the geometric interpretation of the case in which equation (5) is transformed into an identity within certain limits of variation of u and for certain values of the parameters. If such an identity holds, then a line C will be found in the field (u, C) with which the carrier of the dummy scale L will coincide on some segment.

A particular case of equation (1) is the relation

$$v = \frac{Au + B}{Cu + D}.$$

Equations (1) and (2) contain, as important particular cases, dependences representable by nomograms with an oriented tracing sheet and by nomograms of aligned points. The number of parameters for these types of nomograms is reduced by 1.

A. For nomograms with an oriented tracing sheet, equations (1)–(5) take the form:

$$\Phi[v, f(u) + A, g(u) + B] = 0; \tag{6}$$

$$F[f(u, v) + A, g(u, v) + B] = 0; \tag{7}$$

$$F\{f[u, \varphi(u) - \Delta] + A, g[u, \varphi(u) - \Delta] + B\} = 0; \tag{8}$$

$$F\{f[u, (1 - \delta)\varphi(u)] + A, g[u, (1 - \delta)\varphi(u)] + B\} = 0; \tag{9}$$

$$F\{f[u, \varphi(u), C] + A, g[u, \varphi(u), C] + B\} = 0. \tag{10}$$

A nomographic solution of problem 3 is also possible for dependences

$$\varphi(u) - \Delta = A + P(u, B), \quad [(1 - \delta)\varphi(u) = AP(u, B),$$

$$\varphi(u) - \Delta = A + P[g(u) + B], \quad [(1 - \delta)\varphi(u) = AP[g(u) + B],$$

which reduce to the form of equation (6). The variables Δ and δ in this case are represented either by scales or by families of lines (the last two forms).

Important particular cases of the dependences (8), (9), and (10):

$$\varphi(u) - \Delta = P\{u, A + Q[g(u) + B]\}, \quad (1 - \delta)\varphi(u) = P\{u, A + Q[g(u) + B]\},$$

$$\varphi(u) = P\{u, C, A + Q[g(u, C) + B]\}, \quad \varphi(u) = AP(u) + BQ(u, C) + R(u, C).$$

B. In the case of a nomogram of aligned points, equations (2)–(5) are replaced by the following:

$$A + Bf(u, v) + g(u, v) = 0; \tag{11}$$

$$A + Bf[u, \varphi(u) - \Delta] + g[u, \varphi(u) - \Delta] = 0; \tag{12}$$

$$A + Bf[u, (1 - \delta)\varphi(u)] + g[u, (1 - \delta)\varphi(u)] = 0; \tag{13}$$

$$A + Bf[u, \varphi(u), C] + g[u, \varphi(u), C] = 0. \tag{14}$$

Let us note the possibility of another nomographic solution of problems 1 and 2 for the special case of the dependence (11)

$$A + B\,\frac{f_1(u) - f_2(v)}{g_2(v) - g_1(u)} + \frac{f_1(u)g_2(v) - f_2(v)g_1(u)}{g_1(u) - g_2(v)} = 0, \tag{15}$$

which admits a nomogram of aligned points with curvilinear scales u and v and a binary field (A, B). When formula (15) and its special cases are used as interpolation formulas, the binary field (A, B) becomes mute.

A nomographic solution of problem 3 is also possible for dependences

$$\varphi(u) - \Delta = AP(u, B) + Q(u, B), \qquad (1 - \delta)\varphi(u) = AP(u, B) + Q(u, B),$$

which admit nomograms of aligned points in which the variables Δ or δ are represented by scales.

Important special cases of equations (12)–(14):

$$\varphi(u) - \Delta = AP(u) + BQ(u) + R(u), \qquad (1 - \delta)\varphi(u) = AP(u) + BQ(u) + R(u),$$

$$\varphi(u) = AP(u, C) + BQ(u, C) + R(u, C).$$

Practical examples of the solution of problems 2, 3, and 4 by means of nomograms of aligned points may be found in works [1–4].

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Note: Figure translations are in progress. See original paper for figures.