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Abstract

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PHYSICS

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INTERACTION OF TWO BODIES "EMITTING" GAS FLOWS

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Let there be two stationary spherical bodies, of masses M_1 and M_2, located at a distance r_0 from one another. We shall regard the gas emitted by the bodies as ultrarelativistic; let the equation of state of the gas be

$$pv = \bar{R}T, \tag{1}$$

where $\bar{R} = \text{const}$. The adiabatic equation is $pv^{1/3} = \text{const}$, with $p = \frac{1}{3}\rho c^2$. It may be assumed approximately that the pressure of the gas flowing out of one body will fall to some pressure of the external medium, determined likewise by the gas flow issuing from the other body (see Fig. 1). Obviously,

$$r_1^2 = r_2^2 + r_0^2 - 2r_0 r_2 \cos\varphi \cos\lambda, \tag{2}$$

where φ is the "latitude" and λ the "longitude" relative to the second body of a given point located at a distance r_1 from the first body and at a distance r_2 from the second body.

Fig. 1

If $r_0 \gg R_1$ and $r_0 \gg R_2$, where R_1 and R_2 are the radii of the bodies M_1 and M_2, then, as is known [1],

$$r_2^2 = \frac{\bar{m}_2 v_{02}}{c} \left(\frac{p_{02}}{p_{\Pi 2}}\right)^{(2-k)/k}, \tag{3}$$

where $m_2 = \bar{m}_2/4\pi$; $\bar{m}_2$ is the mass of gas emitted by the body M_2; p_{02}, v_{02} are the pressure and specific volume of the gas in the body M_2. Taking $k = 4/3$, we obtain

$$\frac{p_{\Pi 2}}{p_{02}} = \left(\frac{\bar{m}_2 v_{02}}{c r_2^2}\right)^2. \tag{4}$$

Analogously, for the body M_1 we shall have

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$$\frac{p_{\Pi 1}}{p_{01}} = \left(\frac{\bar{m}_1 v_{01}}{c r_1^2}\right)^2. \tag{5}$$

It follows from this that the pressure force, and the proportional reaction force, are $F \sim m^2/r^2$ (if $k \neq 4/3$, then this law will no longer hold).

When the gas may be considered as a continuous medium, outflow from both bodies will continue until the pressures equalize, i.e., until the condition

$$p_{\Pi 1} = p_{\Pi 2}, \tag{6}$$

is satisfied, where $p_{\Pi 1}$ and $p_{\Pi 2}$ are the gas pressures for the first and second bodies, respectively. However, if lateral overflow of gas is taken into account, then outflow will continue until, on each elementary area, there is no equa-

the acting forces change as $\sim \int p\, dr^2$. In this case, on the average a condition of the form must be satisfied

$$p_{\text{p}1} r_1^2 = p_{\text{p}2} r_2^2. \tag{7}$$

If rarefied gases flow out, then the depth of their mutual penetration must be taken into account. In this case the interaction of the bodies will decrease somewhat.

On the basis of (4) and (5) there is the relation $m_1^2 p_{01} v_{01}^2 / r_1^2 = m_2^2 p_{02} v_{02}^2 / r_2^2$, or, on the basis of (1), assuming that the temperature $T_0 = \text{const}$, $\overline{m}_1 \sqrt{v_{01}}/r_1 = \overline{m}_2 \sqrt{v_{02}}/r_2$.

Eliminating r_1^2 from (2) with the aid of (6), and neglecting the lateral overflow of the gas, which is insignificant, we arrive at the equation of the surface on which $p_{\text{p}1} r_1^2 = p_{\text{p}2} r_1^2$: $r_2^2(1-\overline{a}) - 2 r_0 r_2 \cos\varphi \cos\lambda + r_0^2 = 0$, where $\overline{a} = \overline{m}_1^2 v_{01} / \overline{m}_1^2 v_{02}$, whence

$$\frac{r_2}{r_0} = \frac{1}{1-\overline{a}} \left[\cos\varphi \cos\lambda \pm \sqrt{\cos^2\varphi \cos^2\lambda - 1 + \overline{a}}\right] \tag{8}$$

It is known [1] that if $r_0 \gg R$, then the outflow velocity

$$\frac{a_{\text{p}}}{c} = 1 - \frac{1}{2}\left(\frac{p_{\text{p}}}{p_0}\right)^{2(k-1)/k};$$

for the body M_2, at $k = 4/3$, this expression has the form

$$\frac{a_{\text{p}2}}{c} = 1 - \frac{1}{2}\sqrt{\frac{p_{\text{p}2}}{p_{02}}} = 1 - \frac{m_2 v_{02}}{2 c r_2^2}. \tag{9}$$

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Since the mass of gas flowing through an elementary area $df = r^2 \cos\varphi \, d\varphi \, d\lambda$ is $d\overline{m} = \overline{m} \, df/4\pi r^2 = \overline{m} \cos\varphi \, d\varphi \, d\lambda$, the projection of the force acting on the body M_2 onto the axis r_0 is

$$-F = j = \int_0^{\overline{m}_2} a_{x2} \left(\frac{a_{\text{p}2}}{c} + \frac{c}{3a_{\text{p}2}} \right) d\overline{m} = 4m_2 c \int_0^{\pi/2} \int_0^{\pi} \left(\frac{a_{\text{p}2}}{c} + \frac{c}{3a_{\text{p}2}} \right) \cos^2\varphi \cos\lambda \, d\varphi \, d\lambda, \tag{10}$$

where $a_{x2} = a_2 \cos\varphi \cos\lambda$.

Substituting here $a_{\text{p}2}$ from (9), we arrive at the expression $F = \dfrac{{}^4\!/_3 m_2^2 v_{02}}{r_0^2} \overline{F}(\overline{a})$, where

$$\overline{F}(\overline{a}) = \int_0^{\pi/2} \int_0^{\pi} \left[\cos\varphi \cos\lambda + \sqrt{\cos^2\varphi \cos^2\lambda - 1 + \overline{a}} \right]^2 \cos^2\varphi \cos\lambda \, d\varphi \, d\lambda =$$

$$= 4 \int_0^{\pi/2} \int_0^{\pi/2} \sqrt{\cos^2\varphi \cos^2\lambda - 1 + \overline{a}} \, \cos^3\varphi \cos^2\lambda \, d\varphi \, d\lambda. \tag{11}$$

In the general case, it is difficult to compute $\overline{F}(\overline{a})$. If $\overline{a} > 1$, which may occur when $M_1/M_2 > 1$ and, consequently, when $\overline{m}_1/\overline{m}_2 > 1$, then

$$\overline{F}(\overline{a}) \cong \frac{2}{3} \pi \sqrt{\overline{a}} \left(1 - \frac{1}{5\overline{a}} \right). \tag{12}$$

In this case

$$F \cong \frac{m_1 m_2 \sqrt{v_{01} v_{02}}}{18\pi r_0^2} \left[1 - \frac{\overline{m}_2^2 v_{02}}{5\overline{m}_1^2 v_{01}} \right]. \tag{13}$$

If $\overline{a} = 1$, then the following relations will hold:

$$M_1 = M_2 = M; \qquad \overline{m}_1 = \overline{m}_2 = \overline{m}; \qquad v_{01} = v_{02} = v_0;$$

$$\overline{F}(\overline{a}) = \pi/2; \qquad F = \overline{m}^2 v_0 / 24\pi r_0^2. \tag{14}$$

If in (13) we put $\bar{a} = 1$, we obtain

$$F = \frac{2\bar{m}^2 v_0}{45\pi r_0^2}, \tag{15}$$

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which differs little from (14). Thus, in calculating $\overline{F}(\bar{a})$, it is sufficient to restrict oneself to the first term of the expansion in powers of $\bar{a}$ and to assume that the investigation can be carried out by means of an interpolation relation that gives correct results for $\bar{a} \to \infty$ and $\bar{a} = 1$:

$$F = \frac{\bar{m}_1 \bar{m}_2 \sqrt{v_{01} v_{02}}}{18\pi r_0^2} \left[1 - \frac{\bar{m}_2^2 v_{02}}{4\bar{m}_1^2 v_{01}}\right]. \tag{16}$$

For $\bar{a} \gg 1$

$$F = \frac{\bar{m}_1 \bar{m}_2 \sqrt{v_{01} v_{02}}}{18\pi r_0^2}. \tag{17}$$

It is obvious that the force of interaction between the bodies will be an attractive force, since the gas expands nonuniformly, namely, less when flowing into the region inside the bodies. In this case the difference in the momenta of the outflowing gas will lead to an "attraction" of the bodies. A pressure force F_p also acts on the bodies; however, since $F_p \sim pR^2$, while $F \sim pr_0^2$, we have $F_p/F \sim R^2/r_0^2$, whence it follows that the pressure force may be neglected when $R/r_0 \gg 1$.

The case $M_1 = M_2 = M$ is the most interesting and is treated most correctly, since for $a = 1$ the result does not depend on the criteria of equality of pressures or forces. In this case the surface of "interaction" of the flows is a plane perpendicular to r_0 and bisecting r_0.

True, some difference in the numerical coefficient in the law of interaction can be explained by lateral overflow of gases for dense flows and, if for not very dense flows one takes into account the depth of their mutual penetration. Both these factors may somewhat reduce the force of interaction.

Having correctly considered the case when $M_1 = M_2$, it is easy, by the usual methods of potential theory (assuming that a superposition of flows takes place, which is valid at large distances from the bodies), to consider the general case when $M_1 > M_2$, assuming that $\bar{M}_1 = \bar{k} M_2$, where $\bar{k} > 1$. In this case it will naturally turn out that the influence of body M_1 on M_2 is equal to the influence of body M_2 on M_1.

Assuming that

$$\bar{m}_{1,2} \sqrt{v_{01,2}} \left(1 - \frac{1}{4} \frac{\bar{m}_2^2}{\bar{m}_1^2} \frac{v_{02}}{v_{01}}\right)^{1/2} = \beta_{1,2} M_{1,2}, \tag{18}$$

we find

$$F = \frac{\beta_1 \beta_2 M_1 M_2}{18\pi r_0^2} = \frac{G M_1 M_2}{r_0^2}, \tag{19}$$

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where we have taken $\beta_1\beta_2 = 18\pi G = \text{const}$.

We have arrived at a law of interaction between bodies of the form of Newton' s or Coulomb' s law.

If we assume that the amount of gas emitted by the bodies per unit time is proportional to the mass of the bodies, then one must put

$$\overline{m}_1 = \overline{\alpha}_1 M_1, \qquad \overline{m}_2 = \overline{\alpha}_2 M_2, \tag{20}$$

where

$$\overline{\alpha}_1 = \sqrt{\frac{18\pi G\rho_{01}}{1-\frac{1}{4}M_2^2/M_1^2}}, \qquad \overline{\alpha}_2 = \sqrt{\frac{18\pi G\rho_{02}}{1-\frac{1}{4}M_2^2/M_1^2}},$$

where $\rho_{01,2} = 1/v_{01,2}$. Hence

$$\rho_{01,2} = \frac{\overline{\alpha}_{1,2}^2}{18\pi G}\left(1 - \frac{1}{4}\frac{M_2^2}{M_1^2}\right). \tag{21}$$

In the general case one may write

$$\rho_{01,2} = \frac{\overline{\alpha}_{1,2}^2}{18\pi G}\,\theta, \tag{22}$$

where $\theta = \theta_0(1 - \frac{1}{4}M_2^2/M_1^2)$; for $M_2/M_1 = 1$, $\theta = \frac{3}{4}\theta_0$, while for $M_2/M_1 \to 0$, $\theta = 1$. The factor $\theta_0 < 1$ takes into account the degree of mutual penetration of the gas streams during their interaction.

Let us now consider the case in which the bodies emit not gas particles, but quanta with rest mass equal to zero, and let us suppose that the same equations of state are valid for radiation. Eliminating from the equations

$$m = \frac{\overline{m}}{4\pi} = \frac{r^2 a}{v\sqrt{1-a^2/c^2}}, \qquad \frac{w}{\sqrt{1-a^2/c^2}} = w_0 \tag{23}$$

the quantity a, we find that for radiation the expression

$$m = \frac{\overline{m}}{4\pi} = \frac{cw_0 r^2\sqrt{1-w^2/w_0^2}}{wv} = \frac{E_0}{4\pi c^2}, \tag{24}$$

will be valid, where E_0 is the total energy of the radiation.

Since

$$\frac{w}{w_0} = \left(\frac{p}{p_0}\right)^{1/4} = \left(\frac{v_0}{v}\right)^{1/3}, \tag{25}$$

replacing w by p, we arrive at the equation

$$\left(\frac{p}{p_0}\right)^{3/2} - \frac{p}{p_0} + \left(\frac{v_0 E_0}{4\pi c^3 r^2}\right)^2 = 0. \tag{26}$$

At large distances from the bodies

$$\frac{p}{p_0} \simeq \left(\frac{E_0 v_0}{4\pi c^3 r^2}\right)^2, \qquad \frac{w}{w_0} \simeq \frac{1}{r}\sqrt{\frac{v_0 E_0}{4\pi c^3}}, \tag{27}$$

and for the force of interaction we arrive at the previous result.

As was shown earlier ([1]), the solution used has meaning for $\infty > r \geq r_{\min} = r_{\mathrm{cr}}$, where

$$r_{\mathrm{cr}}^2 = 3\sqrt{3}\,m/2\rho_0 c = 3\sqrt{3}\,m/8\pi c\rho_0.$$

Replacing $m = \overline{\alpha}M$, we find that

$$r_{\mathrm{cr}}^2 = 3\sqrt{3}\,\overline{\alpha}M/8\pi\rho_0 c.$$

It is natural to assume that $r_{\mathrm{cr}} = R$, where R is the radius of the radiating body. Then we shall have

$$\overline{\alpha}_{1,2} = \frac{8\pi c\rho_{01,2}R_{1,2}^2}{3\sqrt{3}\,M_{1,2}}. \tag{28}$$

Solving (22) and (28) together, we find

$$\overline{\alpha}_{1,2} = \frac{27\sqrt{3}\,GM_{1,2}}{4c\theta R_{1,2}^2}, \qquad \rho_{01,2} = \frac{243GM_{1,2}^2}{32\pi c^2\theta R_{1,2}^4}. \tag{29}$$

Thus, we have expressed the quantities $\overline{\alpha}_{1,2}$ and $\rho_{01,2}$ in terms of the interaction constant between the bodies, their masses, and their sizes.

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REFERENCES

1. K. P. Stanyukovich, DAN, **119**, No. 2 (1958).

Note: Figure translations are in progress. See original paper for figures.

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