Corresponding Member of the Academy of Sciences of the USSR V. L. GINZBURG

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Abstract

Full Text

Corresponding Member of the Academy of Sciences of the USSR V. L. GINZBURG

CRITICAL CURRENT FOR SUPERCONDUCTING FILMS

The investigation of the behavior of superconducting films in a magnetic field makes it possible, generally speaking, to determine the penetration depth of a weak magnetic field into a superconductor, δ_0. For the same purpose one may use the measurement of the critical current at which superconductivity disappears, since in the theory ([1]) the influence of the field and the influence of the current are taken into account on an equal footing.

If films deposited on a cylindrical surface are used (see ([2]), where earlier literature is also indicated), then the determination of the critical current is apparently even more reliable than the determination of the critical field. In this connection we shall dwell on the calculation of the critical current in somewhat greater detail than in ([1]).

Of greatest interest are thin films with thickness $l \sim 10^{-5} \div 10^{-6}$ cm. When cylinders with a diameter of the order of a millimeter are used as substrates, such films may be regarded as plane, with the cylindrical configuration of the film being taken into account only in the boundary conditions for the field. For a plane film the equations determining the function Ψ and the vector potential $\mathbf{A}$ are as follows ([1]):

$$\frac{d^2\Psi_0}{d\xi^2} = \chi^2\{\Psi_0^3 - \Psi_0 + \Psi_0(a_y^2 + a_z^2)\}; \tag{1}$$

$$\frac{d^2 a_y}{d\xi^2} = \Psi_0^2 a_y, \qquad \frac{d^2 a_z}{d\xi^2} = \Psi_0^2 a_z. \tag{2}$$

Here and below the following notation is used:

$$\xi = \frac{x}{\delta_0}, \qquad \delta_0^2 = \frac{mc^2}{4\pi e^2 \Psi_\infty^2}, \qquad \Psi_0^2 = \frac{\Psi^2}{\Psi_\infty^2}, \tag{3}$$

$$\mathbf{a} = \frac{\mathbf{A}}{\sqrt{2}\, H_{\text{km}}\delta_0}, \qquad \chi = \frac{\sqrt{2}\, e}{\hbar c}\, H_{\text{km}}\delta_0^2, \qquad \mathbf{H} = \text{rot}\,\mathbf{A}, \qquad \mathbf{h} = \frac{\mathbf{H}}{\sqrt{2}\, H_{\text{km}}},$$

Fig. 1

Figure 1: Fig. 1

where H_{km} is the critical magnetic field for the bulk metal.

The film is assumed to be situated in the plane yz in such a way that the x-axis is perpendicular to it; at the boundaries of the film, at $x = 0$ and $x = l$, the condition must be satisfied

$$\xi = 0, \quad \xi = \frac{l}{\delta_0}: \qquad \frac{d\Psi_0}{d\xi} = 0. \tag{4}$$

The total current is directed along the axis of the cylinder (the z-axis); in the same direction an external field $H_0 = \sqrt{2}H_{\text{km}}h_0$ may be applied. The field of the current is directed along the y-axis and at the outer surface of the film is equal to $H_I = \sqrt{2}H_{\text{km}}h_I = 2I/cr$, where I is the total current flowing through the film and r is the radius of the cylinder (absolute units are used). On the inner surface

surface of the cylindrical film (at $\xi = 0$) the current field is zero. Thus, the field in the film $\mathbf{h}(\xi)$ must satisfy the conditions*

$$\xi = 0: \quad h_y = 0,\ h_z = h_0; \qquad \xi = l/\delta_0: \quad h_y = h_I,\ h_z = h_0. \tag{5}$$

If the thickness satisfies the condition

$$\left(\frac{\chi l}{\delta_0}\right)^2 \ll 1, \tag{6}$$

then the function Ψ_0 may be regarded as independent of the coordinates ([1,3]). Under this assumption the field $\mathbf{h}$ and the corresponding potential $\mathbf{a}$, satisfying equations (2) and conditions (5), are as follows:

$$a_y = \frac{h_0}{\Psi_0}\left\{\operatorname{sh}\Psi_0\xi - \frac{\operatorname{ch}\Psi_0\xi}{\operatorname{cth}(\Psi_0 l/2\delta_0)}\right\},$$

$$h_z = \frac{da_y}{d\xi} = h_0\left\{\operatorname{ch}\Psi_0\xi - \frac{\operatorname{sh}\Psi_0\xi}{\operatorname{cth}(\Psi_0 l/2\delta_0)}\right\}; \tag{7}$$

$$a_z = -\frac{h_I \operatorname{ch}\Psi_0\xi}{\Psi_0 \operatorname{sh}(\Psi_0 l/\delta_0)}, \qquad h_y = -\frac{da_z}{d\xi} = \frac{h_I \operatorname{sh}\Psi_0\xi}{\operatorname{sh}(\Psi_0 l/\delta_0)}.$$

Fig. 1

To determine the quantity Ψ_0, we integrate equation (1) over the limits from $\xi = 0$ to $\xi = l/\delta_0$. Then, using conditions (4) and taking into account the constancy of Ψ_0, we obtain:

$$\Psi_0^2 = 1 - \frac{\delta_0}{l} - \int_0^{l/\delta_0} (a_y^2 + a_z^2)\, d\xi. \tag{8}$$

Substituting here the solution (7), we obtain the relation for Ψ_0:

$$(\Psi_0^2 - 1)\Psi_0^2 = -\frac{\left(\frac{H_I}{H_{\text{KM}}}\right)^2 \left\{1 + \frac{\text{sh}(2\Psi_0 l/\delta_0)}{2\Psi_0 l/\delta_0}\right\}}{4\,\text{sh}^2(\Psi_0 l/\delta_0)} + \frac{\left(\frac{H_0}{H_{\text{KM}}}\right)^2 \left\{1 - \frac{\text{sh}(\Psi_0 l/\delta_0)}{\Psi_0 l/\delta_0}\right\}}{4\,\text{ch}^2(\Psi_0 l/2\delta_0)}. \tag{9}$$

This equation has a solution only so long as $H_I \le H_{Ik}$, which is clear, for example, from Fig. 1, corresponding to the value $l/\delta_0 = 1$.

Thus, the critical current field $H_{Ik} = 2I_k/cr$ is determined from the condition $dH_I/d\Psi_0 = 0$. The graph of the function $H_{Ik}(l/\delta_0)$ at $H_0 = 0$ is shown in Fig. 2. In the limiting cases, from (9) it is easy to obtain the expressions (Ψ_{0k} is the value of Ψ_0 at $H_I = H_{Ik}$):

$$\Psi_{0k}\frac{l}{\delta_0} \ll 1: \qquad \frac{H_{Ik}}{H_{\text{KM}}} = \frac{2\sqrt{2}}{3\sqrt{3}}\frac{l}{\delta_0}\left[1 - \left(\frac{H_0}{H_{\text{KM}}}\right)^2 \frac{l^2}{24\delta_0^2}\right]^{3/2}, \tag{10}$$

$$\Psi_{0k} = \sqrt{\frac{2}{3}\left[1 - \left(\frac{H_0}{H_{\text{KM}}}\right)^2 \frac{l^2}{24\delta_0^2}\right]^{1/2}};$$

$$\Psi_{0k}\frac{l}{\delta_0} \gg 1: \qquad \frac{H_{Ik}}{H_{\text{KM}}} = \left[\frac{8}{5}\sqrt{\left(\frac{3}{5}\right)^3 \frac{l}{\delta_0}} - 2\left(\frac{H_0}{H_{\text{KM}}}\right)^2\right]^{1/2}, \qquad \Psi_{0k} = \sqrt{\frac{3}{5}}. \tag{11}$$

* It is assumed that no total current flows in the film perpendicular to the axis of the cylinder. This can be achieved by cooling the film in the given field h_0 from a region above the critical temperature T_k, or by making the film single-connected by means of a cut along a generator of the cylinder.

In the absence of current, the destruction of superconductivity of the film by the field H_0 occurs at the value H_κ, which was calculated in ([1,3]). If in this case $l \le l_\kappa = \sqrt{5}\,\delta_0$, the transition to the normal state has the character of a second-order transition, and

Fig. 2 graph

Figure 2: Fig. 2 graph

$$\frac{H_\kappa}{H_{\kappa\mathrm{m}}} = \frac{\sqrt{24}\,\delta_0}{l}. \tag{12}$$

From (10) and (12) it follows that, for $H_0 = 0$,

$$H_{I\kappa} H_\kappa = {}^8/_3 H_{\kappa\mathrm{m}}^2. \tag{13}$$

Fig. 2. 1—according to formula (9) with $H_0 = 0$; 2—$H_{I\kappa}/H_{\kappa\mathrm{m}} = 0.544\,l/\delta_0$; 3—$H_{I\kappa}/H_{\kappa\mathrm{m}} = 0.86\sqrt{l/\delta_0}$

Let us note that for a flat film (and not a cylinder) relation (9) is obtained with the replacement, in the term proportional to H_I^2, of the thickness l by $l/2$ (see ([1]), where $l = 2d$). Taking this circumstance into account leads to the fact that in the formulas used in ([2]) an additional factor 2 must be included. To test the theory, it is especially convenient to use relation (13), since in this case it is not necessary to determine δ_0 and l independently.

It follows from experimental data that usually, at least for massive specimens, with good accuracy

$$H_{\kappa\mathrm{m}} = H_0 \left[1 - \left(\frac{T}{T_\kappa}\right)^2\right], \qquad \delta_0 = \frac{\delta_{00}}{\sqrt{1-(T/T_\kappa)^4}}. \tag{14}$$

If these expressions are adopted, then from (9) or (10), (11) one obtains a quite definite temperature dependence of the field H_{Ik}. At the same time it should be noted that the specific form of the initial equation (1) is connected with the assumption of a definite dependence of the free energy of the superconductor F_s on Ψ_0^2, namely, the dependence

$$F_s = F_n + \frac{H_{\mathrm{km}}^2}{8\pi}\left(\Psi_0^4 - 2\Psi_0^2\right) = F_n + \alpha\Psi^2 + \beta\frac{\Psi^4}{2}.$$

The question of the grounds for such a choice and the possibility of considering a more general case is discussed in ([4]). We shall therefore restrict ourselves to the remark that near T_k this choice is especially reliable, and for H_{km} and δ we have

$$H_{\mathrm{km}} = \left|\frac{dH_{\mathrm{km}}}{dT}\right|_k \Delta T, \qquad \delta_0 = \frac{\delta'_{00}}{(\Delta T)^{1/2}}, \qquad \Delta T = T_k - T \ll T_k; \tag{15}$$

If expressions (14) are valid, then, obviously,

$$\left|\frac{dH_{\text{km}}}{dT}\right|_k = \frac{2H_0}{T_k}, \qquad \delta'_{00} = \sqrt{\frac{T_k}{4}}\,\delta_{00}.$$

The quantities $|dH_{\text{km}}/dT|_k$ and δ'_{00} are determined especially reliably from experiment.

Near T_k, formula (10) for $H_0 = 0$ takes the form

$$H_{Ik} = \frac{2\sqrt{2}}{3\sqrt{3}}\left|\frac{dH_{\text{km}}}{dT}\right|_k \frac{l}{\delta'_{00}}(\Delta T)^{3/2}. \tag{16}$$

Under conditions in which proportionality of the field H_{Ik} to the quantity $(\Delta T)^{3/2}$ is observed, the use of expression (16) should be a convenient method for finding the coefficient δ'_{00}.

Note added in proof. Recently, in connection with the construction of the microscopic theory of superconductivity (a preprint of the corresponding detailed paper by Bardeen, Cooper, and Schrieffer was received in Moscow at the end of November 1957), it has become clear that the macroscopic theory ([1]) can be strictly valid quantitatively only in certain limiting cases. At the same time, a more general theory suitable for considering the behavior of superconductors in strong fields and, in particular, for calculating the fields H_k and H_{Ik}, has not yet been developed. It may be thought that measurement of the fields H_k and H_{Ik} for thin films will be a convenient method for experimentally identifying the range of applicability of the results following from theory ([1]), and also for checking more general formulas that will be obtained in the future.

Physical Institute named after P. N. Lebedev
Academy of Sciences of the USSR

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Note: Figure translations are in progress. See original paper for figures.