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1958

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Abstract

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ON THE EMERGENCE OF A SUPERFLUID STATE OF THE ATOMIC NUCLEUS

(Presented by Academician N. N. Bogolyubov, 29 VII 1958)

It is known ([1]) that weak interactions of electrons with equal and opposite momenta near the Fermi surface lead to the appearance of superconductivity in a metal. The superconducting state is energetically more favorable than the state of a completely degenerate Fermi gas, which we shall call normal. On the basis of a certain similarity between the Fermi systems of the nucleus and of a metal ([2]), let us consider the interactions of nucleons in complex nuclei and investigate the possibility of the appearance of such a state of the atomic nucleus which will be lower in energy than the normal state. We shall call this state the superfluid state of the atomic nucleus.

We shall investigate the emergence of the superfluid state of the atomic nucleus by means of the variational principle proposed by N. N. Bogolyubov ([3]), which is a generalization of the well-known Fock method ([4]), and by means of the mathematical methods developed in ([1]).

On the basis of the shell model of the nucleus and with medium and heavy nuclei in mind, let us consider weak interactions ([5]) of protons (or neutrons) situated on one and the same shell near the energy of the Fermi surface. We shall characterize the state of a proton by a set of quantum numbers s, determining the shell, and by the quantum number m of the projection of the angular momentum on the axis of symmetry of the nucleus. We assume that the field of the nucleus deviates somewhat from a centrally symmetric one; therefore there will be no energetic degeneracy with respect to m.

Let us consider only that part of the Hamiltonian which describes the interaction of protons situated on one and the same shell, namely:

$$H = \sum_{s,m,\sigma} \{E(s,\sigma m) - E_F\} a^+_{m\sigma}(s) a_{m\sigma}(s) + \tag{1}$$

$$+\frac{1}{2N} \sum_{\substack{s,\sigma_1,\sigma_2,m_1,m_2,m_1',m_2' \\ (m_1+m_2=m_1'+m_2';\ m_1\neq m_1')}} J(s \mid \sigma_1 m_1,\ \sigma_2 m_2;\ \sigma_1 m_1',\ \sigma_2 m_2') a^+_{m_1\sigma_1}(s) a^+_{m_2\sigma_2}(s) a_{m_2'\sigma_2}(s) a_{m_1'\sigma_1}(s),$$

where N is the number of levels; $\sigma = \pm 1$ characterizes the sign of m; the remaining notation is given in ([5]). The function $J(s \mid m_1, m_2;\ m_1', m_2')$ is real and has the following properties:

$$J(s \mid m_1', \; m_2'; \; m_1, \; m_2) = J(s \mid m_1, \; m_2; \; m_1', \; m_2'),$$

$$J(s \mid -m_1, \; -m_2; \; -m_1', \; -m_2') = J(s \mid m_1, \; m_2; \; m_1', \; m_2').$$

Let us perform a canonical transformation of the Fermi amplitudes

$$a_{m,\sigma}(s) = u_m(s)\alpha_{m,-\sigma}(s) + \sigma v_m(s)\alpha^+_{m\sigma}(s), \tag{2}$$

where

$$\eta_m(s) = u_m(s)^2 + v_m(s)^2 - 1 = 0. \tag{3}$$

Let us find the mean value of H in the new vacuum state c_0, $\alpha_{m\sigma}c_0 = 0$, namely:

$$\begin{aligned}\overline{H} = 2\sum_{s,m}\{E(s,m) - E_F\}v_m(s)^2 + \\ + \frac{1}{N}\sum_{\substack{s,m,m' \\ m\neq m'}}\{J(s \mid m, -m; m', -m')u_m(s)v_m(s)u_{m'}(s)v_{m'}(s) - \\ - J(s \mid m, m'; m', m)v_m(s)^2 v_{m'}(s)^2\} \equiv \mathcal{E}(u,v),\end{aligned} \tag{4}$$

where

$$E(s,m) = E(s,-m).$$

We determine u, v from the condition of a minimum of $\mathcal{E}(u,v)$ in the presence of the additional condition (3). We obtain

$$\xi_m(s)u_m(s)v_m(s) + \frac{u_m(s)^2 - v_m(s)^2}{2N}\sum_{m'} J(s \mid m, -m; m', -m')u_{m'}(s)v_{m'}(s) = 0, \tag{5}$$

where

$$\xi_m(s) = E(s,m) - E_F - \frac{1}{N}\sum_{m'} J(s \mid m, m'; m', m)v_{m'}(s)^2. \tag{6}$$

Equation (5) admits the solution $u_m(s) = 1 - \theta_F(s,m)$, $v_m(s) = \theta_F(s,m)$, which corresponds to the normal state. The function $\theta_F(s,m) = 1$ if $E(s,m) < E_F$, and $\theta_F(s,m) = 0$ if $E(s,m) > E_F$.

We introduce a new unknown function

$$C_m(s) = \frac{1}{N}\sum_{m'} J(s \mid m, -m; m', -m'),$$

related to $u_m(s)$ and $v_m(s)$ in the following way:

$$u_m(s)^2 = \frac{1}{2}\left[1 + \frac{\xi_m(s)}{\tilde{\varepsilon}_m(s)}\right], \qquad v_m(s)^2 = \frac{1}{2}\left[1 - \frac{\xi_m(s)}{\tilde{\varepsilon}_m(s)}\right],$$

where

$$\tilde{\varepsilon}_m(s) = \sqrt{C_m(s^2) + \xi_m(s)^2},$$

and we obtain for $C_m(s)$ the equation

$$C_m(s) = -\frac{1}{N}\sum_{m'} J(s \mid m, -m; m', -m')\frac{C_{m'}(s)}{\sqrt{C_{m'}(s)^2 + \xi_{m'}(s)^2}}. \tag{7}$$

Consider the interaction of protons located on the shell $s = s_0$, whose energy $E(s_0, m)$ is close to the energy of the Fermi surface, i.e. $E_F - \Delta \le E(s_0, m) \le E_F + \Delta$, with $\Delta \ll E_F$. Passing in (7) from the sum to an integral, we obtain

$$C_m(s_0) = -\frac{1}{2}\int_{m_1}^{m_2} dm'\, J(s_0 \mid m, -m; m', -m')\frac{C_{m'}(s_0)\rho(m')}{\sqrt{C_{m'}(s_0)^2 + \xi_{m'}(s_0)^2}}, \quad \text{(7')}$$

where $E(s_0, m_1) = E_F - \Delta$, $E(s_0, m_2) = E_F + \Delta$, $E(s_0, m_0) = E_F$.

In order to obtain the asymptotic form of the solution of (7′) for small J, we pass to the approximate equation

$$C_m(s_0) = \rho(m_0)\ln\frac{C_{m_0}(s_0)}{\mu}\, C_{m_0}(s_0)\frac{J(s_0 \mid m, -m; m_0, -m_0)}{\{d\xi_{m'}(s_0)/dm'\}_{m'=m_0}} +$$
$$+\frac{1}{2}\int_{m_1}^{m_2} dm'\, \ln\frac{|E(s_0, m') - E_F|}{\mu}\,\frac{d}{dm'}\left[J(s_0 \mid m, -m; m', -m')\frac{C_{m'}(s_0)\rho(m')}{d\xi_{m'}(s_0)/dm'}\right], \tag{8}$$

which, for small J, asymptotically coincides with (7). The approximate solution of (8) for small $C_m(s_0)$ is found in the following form:

$$C_m(s_0) = \omega \frac{J(s_0 \mid m, -m;\, m_0, -m_0)}{J(s_0 \mid m_0, -m_0;\, m_0, -m_0)} \exp\left[-\frac{\frac{1}{\rho(m_0)} \left\{ \frac{d\xi_{m'}(s_0)}{dm'} \right\}\Big|_{m'=m_0}}{J(s_0 \mid m_0, -m_0;\, m_0, -m_0)} \right], \tag{9}$$

where

$$\ln \frac{\omega}{\mu} = \int_{m_1}^{m_2} dm' \left\{ \frac{d\xi_m(s_0)}{dm} \right\}_{m=m_0} \ln \frac{|E(s_0, m') - E_F|}{\mu} \frac{1}{\rho(m_0)} \times$$
$$\times \frac{d}{dm'} \left[\left(\frac{J(s_0 \mid m_0, -m_0;\, m', -m')}{J(s_0 \mid m_0, -m_0;\, m_0, -m_0)} \right)^2 \frac{\rho(m')}{d\xi_{m'}(s_0)/dm'} \right]. \tag{10}$$

From the calculations carried out it is seen that, in order to obtain the asymptotics of the superfluid state of the atomic nucleus, of the entire interaction $J(s \mid m_1, m_2;\, m_1', m_2')$ of protons lying on one and the same shell, only a small part of it is essential, namely $J(s_0 \mid m, -m;\, m', -m')$, i.e. only the interactions of protons with equal and opposite projections of angular momentum on the symmetry axis of the nucleus are essential. The remaining part of the interactions of protons in the shell should be taken into account by perturbation theory.

Thus, our investigation confirms the regularity of the calculations in [6], carried out with the interaction $J(s \mid m, -m;\, m', -m')$. In [6] it is shown that the energy of the first excited state is separated from the ground superfluid state by a gap

$$\Delta E \sim 2\omega \exp\left[-\frac{\frac{1}{\rho(m_0)} \left\{ \frac{d\bar{E}_{m'}(s_0)}{dm'} \right\}\Big|_{m'=m_0}}{J(s_0 \mid m_0, -m_0;\, m_0, -m_0)} \right] \tag{11}$$

and that the superfluid state, separated from the normal state by a gap, is energetically more favorable.

The investigation presented here, confirming the considerations expressed in [2], indicates that by means of the emergence of a superfluid state of the atomic nucleus it is possible to explain the energy gap between the ground and the first one-particle excited states in heavy even-even nuclei. We note that in odd nuclei there will be no energy gap, and upon excitation transitions within the outer shell will be observed.

In conclusion I express my deep gratitude to Acad. N. N. Bogolyubov for his constant interest in the work and valuable remarks.

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Received
17 VII 1958

REFERENCES

1. N. N. Bogolyubov, V. V. Tolmachev, D. V. Shirkov, *A New Method in the Theory of Superconductivity*, Publishing House of the Academy of Sciences of the USSR, 1958.

2. A. Bohr, B. R. Mottelson, D. Pines, *Phys. Rev.* (in press).

3. N. N. Bogolyubov, DAN, **119**, 244 (1958).

4. V. A. Fock, *Zs. f. Phys.*, **61**, 126 (1930).

5. V. G. Soloviev, DAN, **123**, No. 3 (1958).

6. V. G. Soloviev, ZhETF, **35**, 823 (1958).

Note: Figure translations are in progress. See original paper for figures.