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Abstract

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M. K. POLIVANOV

PROCESSES OF PRODUCTION OF HEAVY MESONS AND HYPERONS FROM THE POINT OF VIEW OF DISPERSION RELATIONS

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1. The interaction of K mesons with nucleons and hyperons is at present studied theoretically by means of perturbation theory ([1]), the Chew method ([2]), and the method of dispersion relations ([3]). We propose to investigate the amplitudes of the processes $\gamma + N \to K + Y$ (photoproduction) and $\pi + N \to K + Y$ (π-production) from the point of view of dispersion relations, which will make it possible partly to clarify the characteristic features of these processes and the role of the various interactions in them.

As is known from the theory of dispersion relations (see, for example, ([4])), the process of photoproduction of K mesons is described by the "retarded" amplitude

$$T^{\mathrm{ret}}_{\alpha\omega}(l) = \int e^{ilx} F^{\mathrm{ret}}_{\alpha\omega}(x)\, dx, \tag{1}$$

where

$$F^{\mathrm{ret}}_{\alpha\omega}(x-y) = e^{i\frac{p-p'}{2}(x+y)} \left\langle p's' \left| \frac{\delta i_\rho(x)}{\delta A_\nu(y)} \right| ps \right\rangle. \tag{2}$$

Here the initial state is described by the momentum and other quantum numbers of the nucleon p, s and of the photon k, ν, and the final state by those of the hyperon p', s' and of the K meson q, ρ, or by the composite indices $\alpha = (p, s, \nu)$ and $\omega = (p', s', \rho)$; $A_\nu(x)$ is the photon field; $\varkappa_\rho(x)$ is the K-meson field.

We shall use the current operators: the K-meson current

$$i_\rho(x) = i\, \frac{\delta S}{\delta \varkappa_\rho(x)}\, S^+$$

and the electromagnetic current

$$I_\nu(x) = i\,\frac{\delta S}{\delta A_\nu(x)}\,S^+.$$

For the amplitude $T^{\rm ret}(l)$ one can write dispersion relations expressing its real part $D_{\alpha\omega}(E)$ through an integral over the energy of the imaginary part $A_{\alpha\omega}(E)$. However, this integral contains experimentally observable quantities only for energies above the threshold value $|E| > E_{\rm thr}$.* The contribution of the "unobservable" region $|E| < E_{\rm thr}$ must be considered separately. Therefore the structure of $A_{\alpha\omega}(E)$ for $|E| < E_{\rm thr}$ plays a major role in describing the process by means of dispersion relations.

2. Let us introduce a certain coordinate system, convenient for obtaining dispersion relations, in which the energies of the initial and final light particles are equal, $k^0 = q^0$. Since all four particles participating in the process have different masses, in order to satisfy the indicated requirement we must use the system $p' + \alpha p = 0$, where

$$\alpha = \left(1 - \frac{\mathfrak{M}^2 - M^2}{\mathbf{p}^2}\right)^{1/2}, \qquad 0 < \alpha < 1$$

($\mathfrak{M}$ is the hyperon mass, M the nucleon mass).

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* We shall not dwell here on the exclusion of the region of negative energies. As is known, this can always be done.

In such a system the reaction threshold is expressed in the form

$$E_{\rm thr} = \alpha|\mathbf{p}| + \frac{\mathfrak{M}^2 - M^2 + m^2}{2(1+\alpha)|\mathbf{p}|}. \tag{3}$$

We shall carry out the study of $A_{\alpha\omega}(E)$ in this coordinate system.

Let us use the following representation for the imaginary part of the amplitude (expansion in a complete system of intermediate states):

$$\begin{aligned} A_{\alpha\omega}(E,\mathbf{p}^2) = (2\pi)^4 i \sum_n \langle p's'|\,j_\phi(0)\,|n\vec{\pi}_1\rangle\langle n\vec{\pi}_1|\,j_\nu(0)\,|ps\rangle \\ \times\,\delta\Big(\sqrt{\mathbf{p}^2+M^2} + E - \sqrt{M_n^2+\vec{\pi}_1^{\,2}}\Big) \\ -\,(2\pi)^4 i \sum_n \langle p's'|\,j_\gamma(0)\,|n\vec{\pi}_2\rangle\langle n\vec{\pi}_2|\,j_\phi(0)\,|ps\rangle \\ \times\,\delta\Big(-\sqrt{\mathbf{p}^2+M^2} + E + \sqrt{M_n^2+\vec{\pi}_2^{\,2}}\Big). \end{aligned} \tag{4}$$

Here the momenta of the intermediate state are exactly determined by the δ-functions,

$$\begin{aligned} \vec{\pi}_1 &= \left(\frac{1-\alpha}{2} - \frac{m^2}{2p^2}\frac{1}{1+\alpha}\right)\mathbf{p} + \lambda\mathbf{e}, \\ \vec{\pi}_2 &= \left(\frac{1-\alpha}{2} - \frac{m^2}{2p^2}\frac{1}{1+\alpha}\right)\mathbf{p} - \lambda\mathbf{e}, \end{aligned} \tag{5}$$

where $\lambda = (E^2 - E_{\mathrm{thr}}^2)^{1/2}$, $\mathbf{e}^2 = 1$, $\mathbf{e}\cdot\mathbf{p} = 0$. Hence, incidentally, it is seen that for $|E| < E_{\mathrm{thr}}$ the momenta of the intermediate state become complex.

Considering products of matrix elements of the currents, we find that the spectrum in the first sum contains the intermediate states

$$N,\ N+\pi, \dots,\ Y+K, \dots, \tag{6}$$

while the spectrum in the second sum contains the states

$$Y,\ Y+\pi, \dots,\ N+\tilde{K}, \dots \tag{7}$$

The one-particle nucleon and hyperon states give the so-called pole contributions, which are calculated exactly. But, beginning with two-particle states—nucleon $+\pi$-meson and hyperon $+\pi$-meson—we have a continuous spectrum.

Indeed, the "mass" of an intermediate state that includes two particles consists of the sum of the particle masses and the kinetic energy of their relative motion, while the conservation laws fix only the sum of the momenta of the two particles. Solving the equations that determine the position of the spectra (the arguments of the δ-functions), we find for the nucleon branch of the spectrum

$$E_n = \frac{M_n^2 - M^2 - \frac{1}{1+\alpha}m^2 - (\alpha+1)\mathbf{p}^2}{2\sqrt{M^2+\mathbf{p}^2}} \tag{8}$$

and for the hyperon branch

$$E_n = -\frac{M_n^2 - M^2 - \frac{\alpha}{1+\alpha}(\mathfrak{M}^2 - M^2 - m^2) - (\alpha+1)\mathbf{p}^2}{2\sqrt{M^2+\mathbf{p}^2}}. \tag{9}$$

Comparing the positions of the poles, the boundaries of the continuous spectra, and the threshold energy E_{thr}, we find that a significant part of the region $|E| < E_{\mathrm{thr}}$ is filled by the continuous spectrum (see Fig. 1).

Fig. 1. Optimal arrangement of the poles (N), (Y) and cut lines $(N+\pi)$, $(Y+\pi)$, corresponding to the minimum threshold energy $E_{\text{thr}} = m$. In this case the arrangements for photoproduction and π-production do not differ qualitatively

Figure 1: Fig. 1. Optimal arrangement of the poles (N), (Y) and cut lines $(N+\pi)$, $(Y+\pi)$, corresponding to the minimum threshold energy $E_{\text{thr}} = m$. In this case the arrangements for photoproduction and π-production do not differ qualitatively

Similarly examining the amplitude of the process $\pi+N \to K+Y$, we see that in this case as well the continuous spectrum from the states $N+\pi$ and $Y+\pi$ enters essentially into the region $|E| < E_{\text{thr}}$. For this case the boundary is determined by the expression

$$E_{\text{thr}} = \left\{ \frac{(1+\alpha)^2}{4}\,\mathbf{p}^2 + \frac{m^2+\mu^2}{2} + \frac{(m^2-\mu^2)^2}{4(1+\alpha)^2\mathbf{p}^2} \right\}^{1/2}, \tag{10}$$

the nucleon branch of the spectrum by the expression

$$E^{(n)} = \frac{M_n^2 - M^2 - E_{\text{thr}}^2 + \left\{ \dfrac{\mathfrak{M}^2 - M^2 - m^2 + \mu^2}{2(1+\alpha)|\mathbf{p}|} \right\}^2 - \mathbf{p}^2}{2\sqrt{M^2+\mathbf{p}^2}}, \tag{11}$$

and the hyperon branch by the expression

$$E^{(n)} = -\frac{M_n^2 - M^2 - E_{\text{thr}}^2 + \left\{ \dfrac{\mathfrak{M}^2 - M^2 + m^2 - \mu^2}{2(1+\alpha)|\mathbf{p}|} \right\}^2 - \mathbf{p}^2}{2\sqrt{M^2+\mathbf{p}^2}}. \tag{12}$$

Fig. 1. Optimal arrangement of the poles (N), (Y) and cut lines $(N+\pi)$, $(Y+\pi)$, corresponding to the minimum threshold energy $E_{\text{thr}} = m$. In this case the arrangements for photoproduction and π-production do not differ qualitatively.

3. The continuous spectrum also enters essentially into the dispersion relations for the scattering of K-mesons on nucleons ([3]). Thus, an inevitable and substantial contribution from continuous-spectrum states proves to be a characteristic feature of all processes with K-mesons and hyperons. The meaning of this fact is obvious. Indeed, π-mesons and nucleons are particles lighter than K-mesons and hyperons, and therefore there is always a combination, allowed by the strangeness-conservation law, which falls into the region $|E| < E_{\text{thr}}$. This is a consequence of an energetically more favorable reaction (for example, $\gamma+N \to \pi+N$), which also proceeds as a virtual one. And since the constant of the π-interaction g,

obviously, is many times larger than the constant of the K-interaction G, the contribution due to these virtual states, proportional to g^2G^2, must be large. Thus, the appearance of a contribution from states with π-mesons indicates that in processes with K-coupling one cannot refrain from considering the π-coupling, but must consider both interactions simultaneously. In particular, this circumstance is unfavorable for applying perturbation theory in G.

4. The nature of the connection between these two interactions can be established more definitely by considering, in the sum (4), the structure of the two-particle terms that give a contribution to the continuous spectrum. Let us, for example, consider the contribution of the state $N + \pi$ to the imaginary part of the amplitude of K-meson photoproduction. This term contains the product of matrix elements

$$\langle Y|t_\rho|N+\pi\rangle\langle N+\pi|I_\nu|N\rangle,$$

where in the intermediate state n there are a nucleon and a π-meson with total momentum π. Representing the nucleon + π-meson state in the form $a_\rho^{(+)}(\vec{\pi}\,')|N\rangle$ and $\langle N|a_{\rho'}^{(-)}(\vec{\pi}\,')$, and commuting the creation and annihilation operators with the current operators, we shall represent the contribution of the two-particle state

through the matrix elements of the retarded radiation operators $\delta i_\rho/\delta\varphi_{\rho'}$ and $\delta l_\nu/\delta\varphi_{\rho'}$*. Such radiation operators, as is known, enter into the amplitudes of the π-meson photoproduction process and of the process $\pi + N \to K + Y$. But only now do the matrix elements correspond to virtual processes below threshold, and therefore complex, rather than real, particle momenta enter into them. We shall call such matrix elements **pseudoamplitudes**. The pseudoamplitudes have the same analytic structure as the amplitudes of real processes and, in principle, may be obtained from them by analytic continuation into the region of complex momenta**. All contributions of two-particle states are expressed in an analogous way through the corresponding pseudoamplitudes. Thus, when described with the aid of dispersion relations, different processes turn out to be connected with one another into a system of relations through an unobservable region. We hope that the presentation given here may prove useful for the application of various approximate methods ([5]) developed on the basis of dispersion relations.

In conclusion I express my deep gratitude to Acad. N. N. Bogolyubov for suggesting the topic and for his constant attention to the work.

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* The connection becomes simpler if in these calculations one passes to the limit of a fixed nucleon. We note that this is now more justified, since here the mass ratio is 7, and not 2.5, as it was in the initial problem.

** After this work was reported at a seminar of the Joint Institute for Nuclear Research, I learned that similar constructions are being developed by R. Goldberger, Nam and Oehme for nucleon–nucleon collisions. I am grateful to Acad. N. N. Bogolyubov for acquainting me with the manuscript of this work.

Note: Figure translations are in progress. See original paper for figures.