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Abstract

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PHYSICS

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SCATTERING OF SOUND WAVES IN IRREGULAR WAVEGUIDES

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1. Until now, the problem of the scattering of sound by small irregularities in a waveguide has been solved only in the first approximation of the method of small perturbations ([1]). In the present work, a generalization of the solution of the problem is given also for the case when the scattered field is not small in comparison with the incident one. Below we consider the case of small fluctuations of the parameters of the medium filling the waveguide, as well as the case of roughness of the waveguide walls.

Let us choose a rectangular cross section of the waveguide and, for simplicity, restrict ourselves to a plane problem; the solution of the three-dimensional problem is carried out in an entirely analogous manner. We shall assume that the left half-waveguide ($x < 0$) contains no irregularities and that the wave is incident from it onto the right half-waveguide ($x > 0$), which contains irregularities. As the incident wave we take the normal wave of number m

$$\varphi_0(x, z) = e^{ik_{mx}x} \cos k_{mz} z. \tag{1}$$

Here and below the notation is taken as in ([1]). We shall seek the scattered field in the form of a superposition of normal waves of the type corresponding to a waveguide without irregularities.

2. Let the refractive index of an inhomogeneous medium filling a waveguide with smooth, absolutely rigid walls fluctuate from point to point near its mean value, equal to unity:

$$n = 1 + \mu(x, z), \qquad |\mu| \ll 1. \tag{2}$$

To calculate the scattered field in the right half-waveguide we use Rytov' s method ([2]), somewhat modified as applied to the conditions of propagation of normal waves in a waveguide. Namely, we use the approximation of Rytov' s method separately for each of the plane waves into which the incident normal wave can be decomposed, requiring at the same time that the total scattered field satisfy the boundary conditions on the walls of the waveguide. In this approximation, the field potential in the right half-waveguide is written in the form

$$\varphi(x,z) = \frac{1}{2}\left\{\exp\left[i\left(\psi_0' + e^{-i\psi_0'}W'\right)\right] + \exp\left[i\left(\psi_0'' + e^{-i\psi_0''}W''\right)\right]\right\}, \tag{3}$$

where

$$\psi_0' = k_{mx}x + k_{mz}z, \qquad \psi_0'' = k_{mx}x - k_{mz}z,$$

$$\begin{aligned} W'(x,z) = \sum_{n=0}^{\infty} \frac{k^2}{\theta_n h k_{nx}} \Bigg\{ & e^{ik_{nx}x} \int_0^x \int_0^h \mu(X,Z) e^{i(k_{mx}-k_{nx})X} [e^{ik_{nz}z}\cos(k_{mz}-k_{nz})Z \\ & + e^{-ik_{nz}z}\cos(k_{mz}+k_{nz})Z]\, dX\, dZ \\ & + e^{-ik_{nx}x} \int_x^{\infty} \int_0^h \mu(X,Z) e^{i(k_{mx}+k_{nx})X} \\ & \times \left[e^{ik_{nz}z}\cos(k_{mz}-k_{nz})Z + e^{-ik_{nz}z}\cos(k_{mz}+k_{nz})Z\right] dX\, dZ \Bigg\}; \end{aligned} \tag{4}$$

$$\begin{aligned} W^n(x,z) = \sum_{n=0}^{\infty} \frac{k^2}{\theta_n h k_{nx}} \Bigg\{ & e^{ik_{nx}x} \int_0^x \int_0^h \mu(X,Z) e^{i(k_{mx}-k_{nx})X} [e^{ik_{nz}z}\cos(k_{mz}+k_{nz})Z + \\ & + e^{-ik_{nz}z}\cos(k_{mz}-k_{nz})Z]\, dX\, dZ + e^{-ik_{nx}x} \int_x^{\infty} \int_0^h \mu(X,Z) e^{i(k_{mx}-k_{nx})X} \times \\ & \times \left[e^{ik_{nz}z}\cos(k_{mz}+k_{nz})Z + e^{-ik_{nz}z}\cos(k_{mz}-k_{nz})Z\right] dX\, dZ \Bigg\}. \end{aligned} \tag{5}$$

Here $\theta_n = 1$ for $n \neq 0$; $\theta_n = 2$ for $n = 0$.

This result has been obtained in the first approximation in μ, under the condition that

$$\frac{1}{k}\left|\nabla\{e^{-i\psi_0'}W'\}\right| \sim |\mu|; \tag{6}$$

$$\frac{1}{k}\left|\nabla\{e^{-i\psi_0''}W''\}\right| \sim |\mu|. \tag{7}$$

In the case of small-scale inhomogeneities, conditions (6) and (7) are satisfied only for a scattered field that is small in comparison with the incident one. This restriction is absent for large-scale inhomogeneities; in this case, in (4) and (5) the second terms in the braces may be discarded. In the remaining infinite sums, the principal importance is possessed by the term with number m and those close to it; therefore, for sufficiently large kh, conditions (6) and (7) will also be satisfied in the case when the scattered field is not small in comparison with the incident field. In what follows, only large-scale inhomogeneities will be considered.

3. Let us now represent the field in the waveguide after passage through an inhomogeneous section of length L by the sum of normal waves of the type corresponding to a waveguide without inhomogeneities. For the statistical function μ we shall calculate the mean squares of the amplitudes of these normal waves in the case when $m = 0$. Denoting the amplitude of the normal wave of number n by A_n, at $x = L$ we have

$$\sum_{n=0}^{\infty} B_n \cos k_{nz}z = \exp\left\{i\left[kL + \sum_{n=0}^{\infty} c_n \cos k_{nz}z\right]\right\}, \tag{8}$$

where

$$B_n(L) = A_n \exp[ik_{nx}L],$$

$$c_n = \frac{2k^2}{\theta_{nhk_{nx}}} \exp[-i(k-k_{nx})L] \int_0^L \int_0^h \mu(X,Z) \exp[i(k-k_{nx})X] \cos k_{nz}Z \, dX \, dZ.$$

Multiplying (8) by $\cos k_{rz}z$ and integrating with respect to z from 0 to h, we obtain

$$B_r = \frac{2}{\theta_r h} \int_0^h \exp\left\{i\left[kL + \sum_{n=0}^{\infty} c_n \cos k_{nz}z\right]\right\} \cos k_{rz}z \, dz. \tag{9}$$

Let us multiply expressions (9) and the complex conjugate of (8), and average the result over the ensemble. For a normal distribution of the quantity μ we obtain

$$\sum_{n=0}^{\infty} \overline{B_{rB}n^*} \cos k_{nz}z = \frac{2}{\theta_r h} \int_0^h e^{-\overline{\eta}^2/2} \cos k_{rz}z_1 \, dz_1, \tag{10}$$

where

$$\eta = \sum_{n=0}^{\infty} \{c_n \cos k_{nz} z_1 - c_n^* \cos k_{nz} z\}.$$

The asterisk denotes the complex-conjugate quantity, and the bar denotes the stati-

statistical averaging. Differentiation of (10) gives

$$\sum_{n=0}^{\infty} k_{nz} \overline{B_r B_n^*} \sin k_{nz} z = \frac{1}{\theta_r h} \int_0^h e^{-\eta^2/2} \frac{\partial \left[\overline{\eta^2}\right]}{\partial z} \cos k_{rz} z_1 \, dz_1. \tag{11}$$

Multiplying (10) by $\cos k_{pz} z$, and (11) by $\sin k_{qz} z$, and integrating with respect to z from 0 to h, we obtain, respectively:

$$\overline{B_r B_p^*} = \frac{4}{\theta_r \theta_p h^2} \int_0^h \int_0^h e^{-\eta^2/2} \cos k_{rz} z_1 \cos k_{pz} z \, dz \, dz_1; \tag{12}$$

$$k_{qz} \overline{B_r B_q^*} = \frac{2}{\theta_r h^2} \int_0^h \int_0^h e^{-\eta^2/2} \frac{\partial \left[\overline{\eta^2}\right]}{\partial z} \cos k_{rz} z_1 \sin k_{qz} z \, dz \, dz_1. \tag{13}$$

Substituting the value of $\overline{\eta^2}$ into (13) and using (12), we find an infinite system of homogeneous algebraic equations for the quantities $\overline{A_n A_l^*}$. Solving this system, one can find the quantities $\overline{A_n A_l^*}$ to within an arbitrary constant factor, determined from the law of conservation of the energy flux in the waveguide.

The mean squares of the amplitudes $\overline{A_n A_n^*}$ can be obtained approximately without solving the entire system of equations. Indeed, putting $r = q$ in the system and discarding small terms, we obtain

$$k_{rz} \overline{A_r A_r^*} = -\frac{1}{4} \sum_{n=0}^{\infty} k_{nz} c_n c_n^* \left[\overline{A_{n+r} A_{n+r}^*} - \overline{A_{|n-r|} A_{|n-r|}^*} \, \theta_{|n-r|}^2\right]. \tag{14}$$

Here r takes arbitrary integer values from 1 to ∞. Solving the system of equations (14) (for example, by the method of successive approximations) and using the relation expressing the law of conservation of the energy flux, we find the quantities $\overline{A_n A_n^*}$.

In Fig. 1 the curves, calculated by the indicated method, are given for the intensities of the scattered normal waves of the first orders as a function of the reduced length of the inhomogeneous section $\xi = \sigma^2 k L$ (solid lines). The calculation was carried out for $m = 0$, $\tau_0 = h$, and $kh = 20.5\pi^*$. For comparison, the

Fig. 1

Figure 1: Fig. 1

intensities obtained by the small-perturbation method (first approximation**) are plotted with dashed lines.

Fig. 1

4. Let the equation of the upper surface in a waveguide with perfectly rigid walls have the form

$$z = h + \zeta(x), \tag{15}$$

where $|k\zeta| \ll 1,\ |d\zeta/dx| \ll 1$.

We seek the solution in the form of the sum of two fields, which satisfy the homogeneous wave equation and, as the roughness tends to zero,

* A noninteger number π was taken in order to exclude transverse resonance.

** The intensity of the zero normal wave was obtained with the aid of normalization according to the law of c

pass into the corresponding plane waves forming (1). In the first approximation of Rytov' s method, the field potential is written in the form (3), where

$$W'(x,z) = \sum_{n=0}^{\infty} \frac{(-1)^{n+m+1/2}}{\theta_n h k_{nx}} e^{ik_{nx}x} \int_0^x \left\{ \left[k_{mx} \frac{d\zeta(X)}{dX} - ik_{mz}^2 \zeta(X) \right] \cos k_{nz} z + \right.$$

$$\left. + k_{nz} k_{mz} \zeta(X) \sin k_{nz} z \right\} e^{i(k_{mx}-k_{nx})X}\, dX; \tag{16}$$

$$W''(x,z) = \sum_{n=0}^{\infty} \frac{(-1)^{n+m+1/2}}{\theta_n h k_{nx}} e^{ik_{nx}x} \int_0^x \left\{ \left[k_{mx} \frac{d\zeta(X)}{dX} - ik_{mz}^2 \zeta(X) \right] \cos k_{nz} z - \right.$$

$$\left. - k_{nz} k_{mz} \zeta(X) \sin k_{nz} z \right\} e^{i(k_{mx}-k_{nx})X}\, dX. \tag{17}$$

This result is obtained under the condition that

$$\frac{1}{k}\left|\nabla\{\varepsilon^{-i\psi_0'}W'\}\right| \sim k|\zeta|; \tag{18}$$

$$\frac{1}{k}\left|\nabla\{\varepsilon^{-i\psi_0''}W''\}\right| \sim k|\zeta|. \tag{19}$$

The scattered field can be represented as a sum of normal waves of the type corresponding to a waveguide without roughness. For the statistical function ζ, the mean squares of the amplitudes of these normal waves are calculated by the method set forth in the preceding section.

In conclusion, I express my deep gratitude to M. A. Isakovich for suggesting the topic and supervising the work, and also to Yu. L. Gazaryan for discussion of a number of questions.

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Note: Figure translations are in progress. See original paper for figures.