On the Analysis of Commutation in Commutator Machines Connected with Contact Conductivity

!\[Fig. 1\](placeholder)

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Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

Abstract

Full Text

Electrical Engineering

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On the Analysis of Commutation in Commutator Machines Connected with Contact Conductivity

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Work in recent years has revealed a number of important phenomena accompanying commutation. Under certain conditions, ionic, electrochemical, thermal, and mechanical phenomena in the brush contact do not play a substantial role. In such practically important cases, which occur in certain types of commutator machines and operating regimes, the commutation processes are connected with the contact conductivity under the brushes ([1–3]). This is the basis for the use, in the theory and design of electrical machines, of the classical theory of commutation, which rests on fundamental investigations and many years of experience ([3–8]). Below an analytical method is set forth for studying commutation under the assumption that it is connected only with the contact conductivity under the brushes.

Fig. 1

Fig. 2

If the resistance of the armature section and of its connecting conductors with the commutator is neglected (Fig. 1), then for the additional commutation current one may obtain

$$i_{\mathrm{d}} R \frac{T}{T-t} + i_{\mathrm{d}} R \frac{T}{t} = -L \frac{d i_{\mathrm{d}}}{dt} + e_{\mathrm{d}}, \tag{1}$$

or

$$\frac{d i_{\mathrm{d}}}{dx} + \frac{A}{x(1-x)} i_{\mathrm{d}} - \frac{A}{R} e_{\mathrm{d}} = 0, \tag{2}$$

where $x = \dfrac{t}{T}$; $A = R\dfrac{T}{L}$; T is the commutation period; R is the resistance of the brush contact.

The commutation equation (2) was first obtained by Arnold and Mie ([9]). Its integral is the function

$$i_\mathrm{d} = \left(\frac{1-x}{x}\right)^A \int_0^x e_\mathrm{d}\frac{A}{R}\left(\frac{x}{1-x}\right)^A dx. \tag{3}$$

If the additional emf e_d generated in the section is constant, then

$$i_\mathrm{d} = e_\mathrm{d}\frac{A}{R}\left(\frac{1-x}{x}\right)^A \int_0^x \left(\frac{x}{1-x}\right)^A dx. \tag{4}$$

In order to obtain the desired analytical solution, let us consider the incomplete beta function

$$\mathrm{B}_x(p,q) = \int_0^x x^{p-1}(1-x)^{q-1}dx. \tag{5}$$

Putting $p-1 = A,\ q-1 = -A$, we obtain

$$\int_0^x \left(\frac{x}{1-x}\right)^A dx = \mathrm{B}_x(1+A, 1-A). \tag{6}$$

Consequently,

$$i_\mathrm{d} = e_\mathrm{d}\frac{A}{R}\frac{\mathrm{B}_x(1+A,1-A)}{\mathrm{B}'_x(1+A,1-A)}, \tag{7}$$

where $\mathrm{B}'_x(1+A,1-A) = d\mathrm{B}_x/dx$.

However, for the ranges of variation $1+A$ and $1-A$, the incomplete beta function has not been tabulated ([10]). Therefore let us consider the ratio

$$F_x(1+A,1-A) = \frac{\mathrm{B}_x(1+A,1-A)}{\mathrm{B}(1+A,1-A)}. \tag{8}$$

Here the complete beta function is

$$\mathrm{B}(1+A,1-A) = \int_0^1 \left(\frac{x}{1-x}\right)^A dx = \Gamma(1+A)\Gamma(1-A). \tag{9}$$

In view of the fact that

$$\frac{d}{dx}\{x^{1+A}(1-x)^{-A}\} = (1+A)x^A(1-x)^{-A} + Ax^{1+A}(1-x)^{-1-A},$$

after integrating this expression we find

$$x^{1+A}(1-x)^{-A} = (1+A)\mathrm{B}_x(1+A, 1-A) + A\mathrm{B}_x(2+A, -A). \tag{10}$$

In exactly the same way, starting from

$$\frac{d}{dx}\{x^{1+A}(1-x)^{1-A}\} = (1+A)x^A(1-x)^{-A} - 2x^{1+A}(1-x)^{-A},$$

we obtain

$$\mathrm{B}_x(1+A, 1-A) = \frac{1}{1+A}x^{1+A}(1-x)^{1-A} + \frac{2}{1+A}\mathrm{B}_x(2+A, 1-A). \tag{11}$$

Passing to the functions (9), we shall have

$$F_x(1+A, 1-A) = \frac{1}{\Gamma(2+A)\Gamma(1-A)}x^{1+A}(1-x)^{1-A} + F_x(2+A, 1-A).$$

Applying the same operation to this expression, we find

$$\begin{aligned} F_x(1+A, 1-A) = {} & \frac{1}{\Gamma(2+A)\Gamma(1-A)}x^{1+A}(1-x)^{1-A}\times \\ & \times\left[1 + \frac{2}{2+A}x + \frac{2\cdot 3}{(2+A)(3+A)}x^2 + \cdots\right. \\ & \left.\cdots + \frac{2\cdot 3\cdots(m-1)m}{(2+A)(3+A)\cdots(m+A)}x^{m-1}\right] + F_n(A+m+1, 1-A). \end{aligned} \tag{12}$$

By majorizing the last term, estimating the remainder, and specifying the accuracy, we determine the required number of terms of the series. In the interval $0 < x < 1/2$ and $0 < A < 1$, the series converges rapidly and is suitable for calculations.

Next, from (12) we find

$$\mathrm{B}_x(1+A, 1-A) = \frac{\Gamma(1+A)}{\Gamma(2+A)}x^{1+A}(1-x)^{1-A}\times$$

$$\times \left[1 + \frac{2}{2+A}x + \frac{2\cdot 3}{(2+A)(3+A)}x^2 + \cdots\right].$$

After transformations, finally:

$$\mathrm{B}_x(1+A,1-A) = x^A(1-x)^{1-A}\sum_{k=1}^{m}\frac{Ak}{A+k}\mathrm{B}(A,k)x^k =$$

$$= x^{1+A}(1-x)^{1-A}\sum_{k=1}^{m}k\mathrm{B}(1+A,k)x^{k-1}.$$

This expression made it possible for us to tabulate the indicated incomplete beta function. In doing so it should be taken into account that $\mathrm{B}_x(1+A,1-A)$ need only be computed in the interval $0 < x < 0.5$. In the interval $0.5 < x < 1$ the values of the function are determined from

$$\int_0^x \left(\frac{x}{1-x}\right)^A dx + \int_x^1 \left(\frac{x}{1-x}\right)^A dx = \Gamma(1+A)\Gamma(1-A),$$

which, after transformations, gives the recurrence relation

$$\mathrm{B}_x(1+A,1-A) + \mathrm{B}_{1-x}(1-A,1+A) = \Gamma(1+A)\Gamma(1-A). \qquad (13)$$

Moreover, taking (10) into account, it is sufficient to compute the function $\mathrm{B}_x(1+A,1-A)$ for $A < 1$, while for $A' = A+1$ the values $\mathrm{B}(1+A',1-A')$ are found from (10).

Thus we arrive at the following expression for the commutation additional current:

$$i_\mathrm{d} = e_\mathrm{d}\frac{A}{R}(1-x)\sum_{k=1}^{m}\mathrm{B}(A+1,k)x^k. \qquad (14)$$

This solution of the classical commutation equation makes it possible to compute the values of the additional current for all practically occurring magnitudes of A.

From (14) there follows the relation

$$\frac{\mathrm{B}_x(1+A,1-A)}{\mathrm{B}'_x(1+A,1-A)} = (1-x)\sum_{k=1}^{m}k\mathrm{B}(1+A,k)x^k, \qquad (15)$$

which is of independent interest.

We have carried out a tabulation of the incomplete beta function for A from 0 to 15, and also of the commutation additional current for the same values of A. It should be noted that dependence (15) is also tabulated by this.

The magnetomotive force of the commutation reaction of the armature is

$$F_{\mathrm{k}} = 2w\beta \int_0^1 i_{\mathrm{d}}\, dx,$$

where w is the reduced number of turns of the armature-winding section; β is the ratio of the brush width to the value of the commutator pitch.

Proceeding from the expressions found above, for the value of the additional current averaged over the commutation period one can obtain

$$i_{\mathrm{dc}} = e_{\mathrm{d}} \frac{A}{R} \sum_{k=1}^{m} \frac{k}{(k+1)(k+2)} \mathrm{B}(1+A, k). \tag{16}$$

The results of computations of the current i_{dc} for $e_{\mathrm{d}} \frac{A}{R} = 1$ are presented in Fig. 2, which can serve directly for calculating the magnetomotive force F_{k} of the commutation reaction of the armature for A from 0 to 10.

If the transformer emf e_t of the commutated section is $e_t = e_{t0} e^{ax}$, then

$$i_{dt} = \left(\frac{x}{1-x}\right)^{-A} \frac{A}{R} \int_0^x e_{t0} e^{ax} \left(\frac{x}{1-x}\right)^{A} dx. \tag{17}$$

Taking into account that in the case under consideration $0 < x < 1,\ ax \ll 1$, the analytical expression i_{dt} can be found by expansion in a series

$$i_{dt} = \frac{Ae_{t0}}{R} x^{-A} (1-x)^{A} \sum_{s=0}^{m} \frac{a^s}{s!} B_x(1+A+s,\, 1-A). \tag{18}$$

After the corresponding transformations we obtain, for this case, an expression suitable for calculations,

$$i_{dt} = i_d \left[1 + \sum_{s=1}^{m} \frac{a^s}{s(s+1)!\, B(1+A,s)}\right] -$$

$$- \frac{Ae_{t0}}{R} \sum_{k,s=1}^{m} \frac{a^s k B(1+A,k)}{s(s+1)!\, B(1+A,s)} (1-x) x^k. \tag{19}$$

Here i_d is determined by (14), in which $e_d = e_{t0}$, or its values may be taken from tables.

For the mean over the commutation period value i^c_{dt} of this component of the current, we find

$$i^c_{dt} = \int_0^1 i_{dt}\,dx = i_{dc}\left[1 + \sum_{s=1}^{m} \frac{a^s}{s(s+1)!\,B(A+1,s)}\right] - $$

$$-\frac{Ae_{t0}}{R}\sum_{s,k=1}^{m} \frac{a^s}{s(s+1)!\,B(A+1,s)}\frac{kB(A+1,k)}{(k+1)(k+2)}. \qquad (20)$$

For $m = 3$ we shall have

$$i^c_{dt} = i_{dc}\left[1 + \frac{a(A+1)}{2} + \frac{a^2(A+1)(A+2)}{12} + \frac{a^3(A+1)(A+2)(A+3)}{144}\right] -$$

$$-\left[\frac{a}{2} + \frac{a^2(A+2)}{12} + \frac{a^3(A+2)(A+3)}{72}\right]x(1-x) -$$

$$-\left[\frac{a^2}{6} + \frac{a^3(A+3)}{72}\right]x^2(1-x) - \frac{a^3}{24}x^3(1-x). \qquad (21)$$

In the sufficiently general case when $e_t = e_{t0}e^{ax}\sin bx$, one may put $\sqrt{a^2+b^2} = r,\ b/r = \sin\varphi,\ a/r = \cos\varphi$ and represent the emf in the form

$$e_t = e_{t0}\sum_{k=1}^{m}\frac{c_k}{k!}x^k;$$

here $c_k = r^k \sin k\varphi$. In this case the preceding result is used directly, with replacement of a^k by c_k.

The results presented show that the solution found for the commutation equation contains broad possibilities for calculations.

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Note: Figure translations are in progress. See original paper for figures.