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PHYSICS

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DISPERSION RELATIONS FOR REACTIONS WITH A VARIABLE NUMBER OF PARTICLES

(Presented by Academician N. N. Bogolyubov on 17 II 1958)

Recently, considerable attention has been devoted to the further development of questions in the theory of dispersion relations. After the investigations of N. N. Bogolyubov ([1]), it became clear that dispersion relations are a consequence of the general principles of local field theory. In this connection, an experimental test of dispersion relations could shed light (in the event of their violation) on the question of the locality of elementary interactions. In addition to the fundamental conclusions that may be obtained from an experimental test of dispersion relations, further development of the method of dispersion relations is of great importance both in the direction of investigating the unobservable region in dispersion relations already obtained and in the direction of extending the method to the study of reactions with a variable number of particles.

An attempt to justify dispersion relations for reactions with a variable number of particles was made in work ([2]). However, the method proposed in that work for justifying dispersion relations is unconvincing and encounters serious objections ([1]).

In the present note (using the process $(\gamma + P + 2\gamma + P)$ as an example), questions in the theory of dispersion relations for reactions with a variable number of particles are considered. These arguments can also be carried over to other reactions. In doing so, we shall not take into account higher-order electromagnetic corrections, focusing attention on the study only of the principal term of the amplitude.

1. Using the causality principle in the form of N. N. Bogolyubov ([3])

$$\frac{\delta j_\nu(x)}{\delta A_{\nu'}(x')} = 0, \qquad x' \lesseqgtr x; \qquad \frac{\delta \Lambda_\nu(x)}{\delta A_{\nu'}(x')} = 0, \qquad x' \gtreqless x, \tag{1}$$

where

$$j_\nu(x) = i\,\frac{\delta S}{\delta A_\nu(x)}\,S^+, \qquad \Lambda_\nu(x) = i\,\frac{\delta S^+}{\delta A_\nu(x)}\,S, \tag{2}$$

and also the properties of translational invariance of matrix elements, we can write the retarded and advanced amplitudes of the double Compton effect in the form

$$T^{\text{ret}}_{\alpha,\omega}(q;q',q'') = -\int e^{i(q'x'+q''x'')}\,dx'\,dx''\,\left\langle p',s'\left|\frac{\delta^2 j_\nu(0)}{\delta A_{\nu'}(x')\,\delta A_{\nu''}(x'')}\right|p,s\right\rangle,$$

$$T^{\text{adv}}_{\alpha,\omega}(q;q',q'') = \int e^{i(q'x'+q''x'')}\,dx'\,dx''\,\left\langle p',s'\left|\frac{\delta^2 \Lambda_\nu(0)}{\delta A_{\nu'}(x')\,\delta A_{\nu''}(x'')}\right|p,s\right\rangle. \tag{3}$$

Here p and q are the momenta of the initial nucleon and photon, and p', q', q'' are the momenta of the final nucleon and photons.

Taking (2) into account, the anti-Hermitian part of the amplitude of the process can be represented in the form

$$A_{\alpha,\omega}(q;q',q'') =$$

$$= -\frac{i}{2}\int e^{i(q'x'+q''x'')}dx'\,dx''\,\left\langle p',s'\left|\frac{\delta^2 j_\nu(0)}{\delta A_{\nu'}(x')\,\delta A_{\nu''}(x'')} + \frac{\delta \Lambda_\nu(0)}{\delta A_{\nu'}(x')\,\delta A_{\nu''}(x'')}\right|p,s\right\rangle. \tag{4}$$

2. According to the law of conservation of the energy-momentum of the system, we have

$$p + q = p' + q' + q''. \tag{5}$$

For simplicity let us consider the case of equal energies ($q'_0 = q''_0$). For this purpose we introduce the vectors Q and Δ, defining them as follows:

$$Q = \frac{1}{2}(q' + q''), \qquad \Delta = \frac{1}{2}(q' - q''), \qquad Q\Delta = 0. \tag{6}$$

Fixing the four-dimensional square of the vector Q,

$$Q^2 = m_Q^2 \tag{7}$$

in the reference frame $p + p' = 0$, we find

$$q = \lambda e - (1+\varepsilon)p, \qquad 2Q = \lambda e + (1-\varepsilon)p, \tag{8}$$

where e is a unit vector orthogonal to the vector p,

$$\varepsilon = \frac{m_Q^2}{p^2}, \qquad \frac{\lambda^2}{4} = E^2 - m_Q^2 - \frac{1}{4}(1-\varepsilon)^2 p^2, \tag{9}$$

and E denotes the zero component of the vector Q.

Dispersion relations are especially interesting when the unobservable energy region is absent. In this case the dispersion relations connect only observable quantities. In the problem under consideration, it is possible (with a suitable choice of the vector $\vec{\Delta}$) to obtain dispersion relations in which the unobservable region is absent.

Let us choose the direction of the vector $\vec{\Delta}$ in such a way that it is orthogonal to the vectors p and e. Then it is easy to see that

$$\Delta_0 = 0, \qquad \vec{\Delta}^2 = m_Q^2. \tag{10}$$

It should be emphasized that in the reference frame chosen by us the amplitude of the process will be a function of the variables E, e, p, Δ, and, when the variable E is varied, the vectors e, p, Δ are fixed.

3. In the dispersion relations connecting the Hermitian and anti-Hermitian parts of the amplitude of the process, there is an unobservable region. The contribution from the unobservable region can be calculated and has the form

$$\frac{1}{\pi}\int_{-E_c}^{E_c} dE' \left(\frac{E-E_0}{E'-E_0}\right)^{n+1} \frac{S_\pm A_{\alpha,\omega}(E')}{(E'-E)} =$$

$$= \sum_{i=1,2}\left[\left(\frac{E_0-E}{E_0+E_i}\right)^{n+1} \frac{S_\pm R_i(p,\vec{\Delta},e)}{E_i+E} + \left(\frac{E_0-E}{E_0-E_i}\right)^{n+1} \frac{S_\pm \Omega_i(p,\vec{\Delta},e)}{E_i-E}\right], \tag{11}$$

where

$$R_1(\mathbf{p},\vec{\Delta},\mathbf{e}) = -\left(1-\frac{E_1}{p^0}\right)\left[V_{q'}(p'',p')D_{q,q''}(p,p'') + V_{q''}(p'',p')D_{q,q'}(p,p'')\right],$$

$$R_2(\mathbf{p},\vec{\Delta},\mathbf{e}) = -\frac{1}{2}\left(1-\frac{2E_2}{p^0}\right)D_{-q',q''}(p'',p')V_q(p,p''), \tag{12}$$

$$\Omega_1(\mathbf{p}, \vec{\Delta}, \mathbf{e}) = -\left(1 - \frac{E_1}{p^0}\right)\left[D_{q,q''}(p'', p')V_{q'}(p, p'') + D_{q,q'}(p'', p')V_{q''}(p, p'')\right],$$

$$\Omega_2(\mathbf{p}, \vec{\Delta}, \mathbf{e}) = -\frac{1}{2}\left(1 - \frac{2E_2}{p^0}\right)V_q(p'', p')D_{-q',q''}(p, p'').$$

Here $S_{\pm}$ is the operator of symmetrization or antisymmetrization with respect to $\mathbf{e}$,

$$E_1 = \frac{\mathbf{p}^2 - m_Q^2}{2p^0}, \qquad E_2 = \frac{\mathbf{p}^2 + m_Q^2}{2p^0}, \qquad E_c = \frac{M\mu + \frac{1}{4}\mu^2 - m_Q^2 - \mathbf{p}^2}{2p^0}.$$

We see that in the unobservable region, in the interval $|E| < E_c$, the one-nucleon state gives a contribution. The continuous spectrum, generally speaking, covers part of the unobservable energy region; however, if the condition $m_Q^2 < \frac{1}{4}\mu^2$ is satisfied, then one can always find an interval of values of $\mathbf{p}$, including the point

$$\mathbf{p}_c^2 = \frac{1}{4}\mu^2 \frac{M}{M + \mu},$$

for which the continuous spectrum lies above the reaction threshold. In the case $m_Q^2 = \frac{1}{4}\mu^2$, the corresponding value of the momentum at which the unobservable region is absent is equal to p_c. The operators V and D enter expression (12). The operator V is the usual 3-vertex operator (it has one photon and two nucleon ends), while the operator D is a 4-vertex operator (it has two nucleon and two photon ends).

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.