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Abstract

Full Text

Physics

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On the Question of Flows Caused by Sound

(Presented by Academician N. N. Andreev, 12 VIII 1958)

The propagation of a sound wave in an absorbing medium may be accompanied by a steady flow (acoustic wind) [(1)], which is sometimes also of practical interest [(2)].

The acoustic wind is usually found by solving the equations of hydrodynamics in the second approximation, taking as the first approximation the solution of the equations of linear acoustics [(3‒6)]. The results obtained in these works, however, are applicable only in the region of small Reynolds numbers $\mathrm{Re} = v_0\lambda\rho_0/b$, where v_0 is the amplitude of the oscillatory velocity; λ is the wavelength; ρ_0 is the equilibrium density; $b = \frac{4}{3}\eta + \zeta$; η, ζ are the coefficients of shear and bulk viscosity. The point is that for $\mathrm{Re} \gg 1$ the wave assumes a sawtooth form as a result of nonlinear effects [(7‒9)], and in this case the solution of the equations of the second approximation becomes inapplicable if the solution of linear acoustics in the form of a harmonic wave is used as the first approximation.

Fig. 1. 1 ‒emitter, 2 ‒boundary of the sound beam, 3 ‒side wall of the tube, 4 ‒axis of the tube

Let us consider the acoustic flow at $\mathrm{Re} \gg 1$ caused by a wave of sawtooth form. Let the emitter be located at a distance l from the beginning of a tube of radius r_0, the ends of which are closed by water-impermeable films transparent to sound (Fig. 1).

Neglecting beam divergence, we shall assume that there is a sound beam of radius r_1*, and that along the path l the form of the wave changes from sinusoidal to sawtooth**. We shall be interested in the flow in the middle section of the tube, while neglecting the influence of its ends. Considering the established motion, we take the Navier‒Stokes equation in the form

$$\langle \nabla \times \nabla \times \nabla \times \mathbf{v} \rangle = -\langle \eta^{-1} \nabla \times \nabla \rho \mathbf{v}\mathbf{v} \rangle, \tag{1}$$

where $\langle\ \rangle$ denotes averaging over time, and $\nabla\rho\mathbf{vv}$ is a vector whose i-th component is equal to $(\partial/\partial x_k)\rho v_i v_k$.

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* A similar problem for $\mathrm{Re} \ll 1$ was considered in (4).

** For this it is necessary that $l \geq c_0^2[\varepsilon\omega v_0]^{-1}$, where $\varepsilon = \frac{1}{2}\frac{\rho_0}{c_0^2}\left(\frac{\partial c^2}{\partial\rho}\right)_s + 1$, c is the speed of sound, and ω is the angular frequency.

We seek an approximate solution of equation (1) in the form

$$\mathbf{v} = \frac{b\omega U(r)}{\rho_0 c_0 \varepsilon}\sum_{n=1}^{\infty}\frac{\sin n(\omega t - kz)}{\operatorname{sh} n(a_0 + az)}\,\mathbf{e}_z + \mathbf{v}_2(r, z), \tag{2}$$

where $U(r)$ is the distribution of the amplitude over the radius; α_0 is a parameter connected with the intensity of the wave at $z = 0$.

The first term on the right-hand side of (2), for $U = \mathrm{const}$, is a solution of the hydrodynamic equations, accurate up to and including terms of second order, describing the propagation of a plane sawtooth wave in an unbounded medium (10). The second term is the time-independent part of the velocity of second order of smallness.

Substituting (2) into (1) and introducing $\mathbf{R} = \langle\nabla\times\mathbf{v}_2\rangle$, we obtain

$$\nabla^2\mathbf{R} = \rho_0\eta^{-1}\left(\frac{b\omega}{\rho_0 c_0\varepsilon}\right)^2\frac{\partial U^2}{\partial r}\sum_{n=1}^{\infty}\frac{n\alpha\operatorname{ch} n(a_0 + az)}{\operatorname{sh}^3 n(a_0 + az)}\,\mathbf{e}_\varphi, \tag{3}$$

which approximately may be rewritten as

$$\nabla^2\mathbf{R} = B\frac{\partial U^2}{\partial r}\,\mathbf{e}_\varphi, \tag{4}$$

where

$$B = \rho_0\eta^{-1}\left(\frac{b\omega}{\rho_0 c_0\varepsilon}\right)^2\sum_{n=1}^{\infty}\frac{n\alpha\operatorname{ch} n\alpha_0}{\operatorname{sh}^3 n\alpha_0}.$$

Equation (4) coincides in structure with formula (32) of (4). Therefore, on the basis of (4), one may immediately write a solution satisfying the boundary condition on the lateral surface of the tube and the condition that the total flux through its cross section be zero.

Putting $U(r) = 1$ for $r \leq r_1$; $U(r) = 0$ for $r > r_1$, we find that $\mathbf{v}_2$ has only a z-component, whose magnitude is equal to

$$v_2 = G_1 \left[\frac{1}{2} \left(1 - \frac{x^2}{y^2} \right) - \left(1 - \frac{1}{2} y^2 \right) (1 - x^2) - \ln y \right], \quad 0 \le x \le y,$$

$$v_2 = -G_1 \left[\left(1 - \frac{1}{2} y^2 \right) (1 - x^2) + \ln x \right], \quad y \le x < 1, \tag{5}$$

where

$$x = \frac{r}{r_0}, \qquad y = \frac{r_1}{r_0}, \qquad G_1 = \frac{b c_0}{\eta \varepsilon^2} r_1^2 \alpha^2 \sum_{n=1}^{\infty} \frac{n \operatorname{ch} n\alpha_0}{\operatorname{sh}^3 n\alpha_0}.$$

The velocity distribution over the radius given by (5) is presented in Fig. 1.

For Re $\ll 1$,

$$\sum_{n=1}^{\infty} \frac{n \operatorname{ch} n\alpha_0}{h^3 n\alpha_0} \simeq \left(\frac{\rho_0 v_0 c_0 \varepsilon}{b\omega} \right)^2,$$

and then

$$G_1 = \frac{1}{2} \alpha \frac{\rho_0 v_0^2 r_1^2}{\eta},$$

which coincides with the result (4). The velocity $\mathbf{v}_2$ was obtained from the value of its curl $\nabla \times \mathbf{v}_2$ under the assumption $\nabla \cdot \mathbf{v}_2 = 0$. It can be shown (5) that the velocity found in this way is the velocity in Lagrangian variables.

The latter circumstance is essential, for, as Rayleigh had already noted, it is precisely the mean value of the velocity in Lagrangian variables that gives the magnitude of the streaming velocity, whereas in Eulerian variables the presence of an acoustic wind is determined by the value of $\langle \rho \mathbf{v} \rangle$. The solution shows that the streaming velocity for Re $\gg 1$ increases with increasing coefficient of absorption of the wave and with its intensity, and decreases with increasing shear viscosity, but the dependence on these quantities has a more complicated character,

than for Re $\ll 1$. It should be noted that solution (5) may lose its meaning as a result of turbulization of the flow.

In conclusion, I express my gratitude to N. N. Andreev for his constant attention and interest in the work, and to Z. A. Goldberg and A. L. Polyakova for discussion of the results.

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Note: Figure translations are in progress. See original paper for figures.