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Abstract

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Small Projective Transformations of a Nomogram

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In constructing nomograms from aligned points for equations with three variables, two of the three scales are usually taken to be equally stretched along the side of the rectangular drawing frame. If such a condition is imposed in advance, then the technique of constructing the first sketch of the nomogram and of subsequently improving it is considerably simplified. The further transformation of the nomogram will consist of a proper projective transformation with parallel invariant straight lines on which two sides of the drawing frame lie, and an affine transformation. But here the question arises: is it not possible to improve the nomogram by compressing one of its scales while preserving the dimensions of the drawing frame? Below we give sufficient conditions under which the nomogram must be constructed with two scales equally stretched along the drawing frame, since compression of one of the scales leads to an increase in the error of computation by the nomogram.

Place the ends of one of the scales of the nomogram at the points $(0;0)$ and $(0;1)$, and the ends of the second scale at the points $(1;0)$ and $(1;1)$. We shall assume that the nomogram lies entirely between the straight lines $y=0$ and $y=1$. Consider a projective transformation under which the point $(1;1)$ goes into the point $(1;1-\alpha)$, where $\alpha>0$, while the other three points remain invariant. It has the form:

$$x_1 = \frac{x}{(x-1)\alpha+1}, \qquad y_1 = \frac{y(1-\alpha)}{(x-1)\alpha+1}. \tag{1}$$

We shall henceforth consider α small. Then, restricting ourselves to terms of the first degree with respect to α, from (1) we obtain

$$x_1 = x[1+(1-x)\alpha], \qquad y_1 = y(1-x\alpha).$$

Under the transformation (1), the linear element ds will, with the same degree of accuracy, pass into ds_1:

$$ds_1^2 = ds^2 + 2\alpha\{(1-2x)\,dx^2 - x\,dy^2 - y\,dx\,dy\}.$$

We shall judge the compression or stretching in individual parts of the square as a result of (1) by the value of the ratio $ds_1 : ds$:

$$\frac{ds_1}{ds} = 1 + \alpha \frac{1 - 2x - yk - xk^2}{1 + k^2} \equiv 1 + \alpha z(x, y, k),$$

where k is the slope of the transformed element ds.

It follows from this that the character of the deformation of the scales of the nomogram will depend on the position of the point of the scale and on the direction of the tangent to the carrier of the scale at this point. For the lines $z(x, y, k) = 0$, the magnitude of ds will be preserved. Replacing k by y', we obtain the differential equation of the lines $ds = \text{const}$:

$$xy'^2 + yy' + 2x - 1 = 0. \tag{2}$$

In Fig. 1 the family of integral curves of equation (2) is shown inside the unit square. The domain of existence of solutions is bounded by the hyperbola $y - 4x(2x - 1) = 0$. The drawing shows only part of its right branch. Through each point inside the domain pass two curves. If k_1 and k_2 are their slopes, then in directions with slope k there will be stretching if $k_1 > k > k_2$, and compression if $k > k_1$ or $k < k_2$. For points outside the domain of existence of solutions, compression will occur in every direction.

If, when transformation (1) is performed, the elements of all the scales of the nomogram fall in regions of compression, then the transformation worsens the nomogram.

Fig. 1 *Fig. 2*

This follows from (1). Under compression, the values of the characteristics of the scales of the nomogram that enter into the expression for the error of computation by the nomogram will decrease, and the magnitude of the error will increase.

Figure 2 gives a sketch of a nomogram with scales p, q, and z. Let us apply to it a transformation under which the equality of the lengths of the scales p and q is violated. The nomogram can be superposed on the unit square in four different ways, aligning the scales p and q with sides parallel to the Oy axis. As a result of carrying out this operation it is found that reducing the scale q leads to compression of the scale z in all its parts; reducing the scale p leads to compression of the scale z on the interval $1.6 < z < 6.6$ and to stretching on the interval $0.8 < z < 1.6$. It follows from this that if the scale z of the nomogram had limits $1.6 < z < 6.6$, then a further improvement of the nomogram would be possible only by means of a transformation under which the parallel straight lines passing through the ends of the scales p and q remain invariant. In the present case, a more careful investigation is necessary of the influence of reducing the scale z under transformation (1) on the magnitude of the error, the expression for which is given in ([1]).

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References Cited

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Note: Figure translations are in progress. See original paper for figures.