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Abstract

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THE LONG-PERIOD LUNI-SOLAR TIDE IN THE WORLD OCEAN

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The expansion of the potential of the tide-generating force of the Moon and the Sun gives, as is known, a series of long-period terms. The most significant of these are the terms of semimonthly, monthly, and semiannual periodicity. In A. Doodson' s expansion these terms are given by the following "argument numbers." Term 075 555 expresses the lunar semimonthly variation of the force potential; term 073 555, the luni-solar semimonthly synodic variation; term 065 455, the monthly variation of the potential of the tide-generating force of the Moon; and, finally, term 057 555, the semiannual variation of the potential of the tide-generating force of the Sun. These terms of the force potential correspond to the long-period tidal waves M_f, M_{sf}, M_m, and S_{sa}.

In the general case the height of the static tide, as is known, is given by the equation

$$H = (1 + k - h)\frac{W}{g},$$

where W is the force potential; g is the acceleration of gravity; the factor $1+k-h$ expresses the elastic properties of the Earth and the influence of the deformation of the body of the Earth on its potential. Calculation of this factor by theoretical means, as is known, is extremely difficult; therefore, in solving such problems, one usually uses values of the factors found from observations of changes in the direction of the plumb line. Taking $h = 0.60$ and $k = 0.27$, we obtain $1 + k - h = 0.67$. The quantities used here were found empirically and are very close to the values calculated by M. S. Molodenskii for the case of a liquid core of the Earth. Then the height of the static tide may be found from the equation:

$$H = 0.67\frac{W}{g}.$$

Using A. Doodson's expansion ([1]), the values of the force potential corresponding to each constituent of the fully long-period tide may be calculated from the equations

$$M_f = 075\,555 = 0.07821 G_0 (1 - 3\sin^2\varphi)\cos 2s;$$
$$M_{sf} = 073\,555 = 0.00685 G_0 (1 - 3\sin^2\varphi)\cos(2s - 2h);$$
$$M_m = 065\,455 = 0.04127 G_0 (1 - 3\sin^2\varphi)\cos(s - p);$$
$$S_{sa} = 057\,555 = 0.3644\, G_0 (1 - 3\sin^2\varphi)\cos 2h.$$

The "geodetic coefficient" used in these equations is determined, according to A. Doodson, by the equality

$$G_0 = \frac{3}{4}\frac{M}{E}\frac{ga^2\rho^2}{c^3},$$

where M is the mass of the Moon; E is the mass of the Earth; g is the mean value of the acceleration of gravity; a is the mean value of the Earth' s radius; ρ is the radius of the Earth at the given point; and c is the mean distance between the centers of the Earth and the Moon.

The numerical value of the geodetic coefficient, for $a = \rho = 6371.2$ km, is, for the potential of the tide-generating force of the Moon,

$$G_0 = 26160 \text{ cm}^2/\text{sec}^2.$$

The relations given make it possible to compute theoretically the height of the static long-period tide in different latitudinal zones of the Earth (see Table 1).

Let us now consider how the long-period tide actually manifests itself in the oceans of the Earth. For this purpose we shall use the data of harmonic analysis of annual cycles of observations of sea-level oscillations ([2]).

Table 1

Heights of long-period tidal waves (in mm)

Wave	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°
M_f	13.9	12.7	9.1	3.5	−3.4	−10.6	−17.5	−23.0	−26.7	−27.9
M_{sf}	1.2	1.1	0.8	0.3	−0.3	−0.9	−1.5	−2.0	−2.3	−2.5
M_m	7.6	6.7	4.8	1.8	−1.8	−5.6	−9.2	−12.2	−14.1	−14.7
$M_f + M_{sf} + M_m$	22.7	20.5	14.7	5.6	−5.5	−17.1	−28.2	−37.2	−43.1	−45.1
S_{sa}	6.5	5.9	4.2	1.6	−1.6	−5.0	−8.1	−10.7	−12.4	−13.0

In general, for the characterization of the fortnightly and monthly tidal waves, data were used from the analysis of a 151-year cycle of observations of tides carried out at various points on the shores of the World Ocean, and for the characterization of the semiannual tide—data from the analysis of a 251-year cycle. The half-amplitudes and phase angles of the long-period tidal waves were distributed by latitude and averaged (see Table 2).

Table 2

Mean harmonic characteristic of long-period tidal waves*

Latitude	M_f a	M_f φ	M_f φ_{av}	M_{sf} a	M_{sf} φ	M_{sf} φ_{av}	M_m a	M_m φ	M_m φ_{av}	S_{sa} a	S_{sa} φ	S_{sa} φ_{av}
60—70° N	34.3	6.2		24.4	10.1		28.1	17.0		52.8	3.5	
50—60°	37.7	9.9	8.5	28.4	11.6	9.4	43.1	17.0	16.5	41.3	3.1	3.1
40—50°	15.2	8.2		21.5	9.4		21.9	15.3		37.3	2.5	
30—40°	14.0	6.5		4.3	5.4		17.1	17.3		44.1	3.5	
20—30°	17.0	1.9		28.9	1.6		22.4	1.3		49.[[unclear: digit]]	0.8	
10—20°	22.1	0.5		22.2	1.5		22.3	0.2		60.[[unclear: digit]]	1.1	
0—10° N	13.4	0.1	0.8	9.7	−2.6	1.1	10.0	1.6	0.6	56.6	1.2	1.4
0—10° S	—	—		—	—		—	—		—	—	
10—20°	39.6	−1.0		—	—		—	—		31.7	1.4	
20—30°	14.3	−1.8		4.6	5.3		20.8	−2.4		18.4	0.9	
30—40°	14.4	5.7	6.7	25.9	10.6	11.2	27.7	15.1	13.5	42.6	2.5	2.7

Latitude	M_f a	M_f φ	M_f φ_{av}	M_{sf} a	M_{sf} φ	M_{sf} φ_{av}	M_m a	M_m φ	M_m φ_{av}	S_{sa} a	S_{sa} φ	S_{sa} φ_{av}
40–50° S	22.3	7.6		27.7	11.8		22.7	11.8		29.1	2.9	

* a is the mean value of the wave half-amplitude in mm; φ is the mean value of the wave phase in days; φ_{av} is the mean weighted value of the wave phase in days. For the wave S_{sa}, the values of the wave phase are given in months.

The data presented make it possible to draw certain new conclusions about the dimensions and character of the real long-period tide in the World Ocean:

1. The total long-period tide in the World Ocean is represented by a two-nodal standing wave. The nodes of this wave are located near 30° north and south latitude, and the antinodes are at the equator and in the polar regions of the ocean. In the equatorial part of the Earth, positive anomalies in the values of the exciting force correspond to the highest position of sea level. In the polar regions of the Earth, on the contrary, positive anomalies of the exciting force coincide in time with negative anomalies in the position of the level. The observational data thus confirm the conclusions of the theory of the long-period tide in the sea.

2. The wave of the long-period tide is a forced wave; however, the phases of this wave do not correspond to the phase values of the force. For all components of the long-period tide waves, observations showed a lag of the wave phase relative to the phase of the force. The mean value of the difference "phase of the level minus phase of the force" proved to be: for the lunar fortnightly wave M_f, +1.1 days; for the lunisolar fortnightly synodic wave M_{sf}, +1.9 days; for the lunar monthly wave M_m, +1.6 days; and, finally, for the solar semiannual wave S_{sa}, +0.3 month. Thus, the mean magnitude of this phase shift for the fortnightly and monthly waves of the long-period tide proved to be 1.5 days. This conclusion is not unexpected, since G. Lamb [3], in the dynamical theory of the long-period tide, pointed out that the deflection corresponding to this type of oscillation depends not only on the perturbing force; as a result, phase shifts arise between the oscillations and the force exciting them, depending on the frequency of the oscillation itself.

3. It was found that the sizes of the long-period tide waves in all cases considerably exceed the sizes predicted for these waves by the static theory of the long-period tide. In the northern hemisphere, in the zone best provided with observations—between 40 and 60° north latitude—the mean values (averaged over 53 cases for the fortnightly and monthly waves and over 86 cases for the solar semiannual wave) of the amplitudes of the individual

waves of the long-period tide were found to be: for the wave M_f, 26.4 mm; for the wave M_{sf}, 25.0 mm; for the wave M_m, 32.5 mm; and for the wave S_{sa}, 39.3 mm. Thus, the fortnightly and monthly waves together can change the position of the mean sea level at these latitudes by 168 mm. In the seas of the Earth' s high latitudes this quantity evidently reaches substantially larger values. The static theory of the long-period tide gives for this characteristic of the height of the combined fortnightly and monthly tides a value equal to only 34 mm. Thus, according to the observational data, the magnitude of the real long-period tide in the middle latitudes of the Earth proved to be at least 5 times greater than the theoretically determined value.

What has been said means that the long-period tide manifests itself in the World Ocean as a fairly powerful phenomenon, capable, by creating astronomic currents of considerable speed, of substantially affecting the circulation of waters and the drift of ice in the seas of the Earth' s high latitudes.

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REFERENCES CITED

[1] A. T. Doodson, Proc. Roy. Soc., A, 100, No. A–704 (1921). [2] P. Schureman, *A manual of the Harmonic Analysis and Prediction of Tide*, Washington, 1924. [3] G. Lamb, *Hydrodynamics*, Moscow–Leningrad, 1947.

Note: Figure translations are in progress. See original paper for figures.