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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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PHYSICS

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RADIATION OF A POINT CHARGE MOVING UNIFORMLY ALONG THE AXIS OF A CIRCULAR APERTURE IN AN INFINITE IDEALLY CONDUCTING PLANE

(Presented by Academician M. A. Leontovich on 7 VII 1958)

Let us consider the radiation of a point charge e_0, moving uniformly with velocity v along the axis of a circular aperture of radius r_0 in an infinite ideally conducting plane. We shall assume $v/c \ll 1$ (c is the speed of light). We place the ideally conducting plane with the aperture in the plane $z = 0$ (Fig. 1). For definiteness, we shall seek the solution for $z \geq 0$.

Fig. 1

To determine $\mathbf{E}$ in the wave zone it is necessary to solve the inhomogeneous wave equation with inhomogeneous boundary conditions. $\mathbf{E}$ may be sought in the form

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2,$$

where $\mathbf{E}_1$ is the solution of the inhomogeneous equation with homogeneous boundary conditions; $\mathbf{E}_2$ is the solution of the homogeneous equation with inhomogeneous boundary conditions.

The first problem reduces to determining the radiation field of a point charge e_0 appearing in the plane $z = 0$ and then moving with constant velocity v along the z-axis. In this case $E_t = 0$ on the entire plane. This is the so-called "transition radiation" when a charge passes from a metal into vacuum. In work ([1]) the radiation field was calculated for the reverse transition (vacuum–metal). For $v/c \ll 1$ the direction of motion (from the metal or into the metal) is immaterial.

Thus, the solution of the first problem has the form

$$\mathbf{E}_1(\omega) = -\frac{e_0 v \sin\vartheta}{\pi c^2}\,\frac{e^{-jkR_0}}{R_0}\,\vec{\vartheta}_0, \tag{1}$$

where e_0 is the magnitude of the charge; v is its velocity; ϑ is the angle between the z-axis and the direction to the observation point; c is the speed of light; $\vec{\vartheta}_0$ is a unit vector directed toward increasing ϑ; k is the wave number; R_0 is the distance from the center of the aperture to the observation point.

To solve the second problem, it is necessary to find the radiation field from the known distribution of the tangential components of the field on the plane $z = 0$. The fields on the surface are specified as follows: $E_t = 0$ for $r > r_0$; $H_t = H_t^0$ for $r < r_0$ (H_t^0 is the field in the case when there is no conducting screen). This is a mixed boundary-value problem, and its solution is rather complicated. However, using the fact that $v/c \ll 1$, we may assume that the electric

the field of the moving charge coincides with the field of a stationary charge placed at the same point at which the moving charge is found at the given instant, and equal to it in magnitude. Then the electric field at the aperture can be calculated by using the method set forth in (2).

Thus our problem can be reduced to the first boundary-value problem, whose solution for fields harmonic in time has the form (3)

$$\mathbf{E}_2(M) = \int_S \mathbf{E}''(M, P, \mathbf{A}^1)\,dS, \tag{2}$$

where $\mathbf{E}_2(M)$ is the field of interest to us at the point M; $\mathbf{E}''(M, P, \mathbf{A}^1)$ is the field produced at the point M by a point magnetic dipole of strength

$$\mathbf{A}^1 = \frac{1}{4\pi}[\mathbf{E}\mathbf{n}],$$

located at the point P on the surface S, with S regarded as perfectly conducting. In our case the integrand differs from zero only at the aperture. Taking this into account, in the wave zone (far from the aperture) (2) can be transformed into the form

$$\mathbf{E}_2(M) = -\frac{j}{\lambda}\,\vec{\vartheta}_0\,\frac{e^{-jkR_0}}{R_0}\int_0^{r_0}\int_{-\pi}^{\pi}\cos\varphi\,E_r(\omega)e^{jkr\cos\varphi\sin\vartheta}\,r\,dr\,d\varphi. \tag{3}$$

Here φ is the angle between the direction from the center of the aperture to the projection of the point M onto the plane $z = 0$ and the direction to the point of integration; $E_r(\omega)$ is the spectral density of E_t at the aperture. By virtue of axial symmetry $E_t = E_r$.

Using (2), we obtain the following expression for E_r, produced by a point static charge e_0 placed on the axis of the aperture at a height z_0 above it:

$$E_r(P) = \frac{2e_0 r}{\pi(r^2+z_0^2)} \left\{ \frac{1}{\sqrt{r^2+z_0^2}} \operatorname{arctg} \sqrt{\frac{r_0^2-r^2}{z_0^2+r^2}} + \frac{1}{\sqrt{r_0^2-r^2}} \right\}, \tag{4}$$

where r is the distance of the point P from the center of the aperture.

E_r is an even function of z_0. Consequently, the radiation of the aperture, as well as the radiation of the charge, does not depend on the direction of motion of the charge,

$$E_r(\omega) = \frac{e_0}{\pi v} \left\{ \frac{r_0 e^{-\omega r_0/v}}{r\sqrt{r_0^2-r^2}} + \frac{\omega}{vr} e^{-\omega r_0/v} \sqrt{r_0^2-r^2} + \frac{\omega^2}{v^2 r} \int_r^{r_0} \sqrt{y^2-r^2}\, e^{-\omega y/v}\, dy \right\}; \tag{5}$$

$$\mathbf{E}_2(M) = \frac{2e_0}{v\lambda} \frac{e^{-jkR_0}}{R_0} \vec{\vartheta}_0 \int_0^{r_0} \left\{ \frac{r_0 e^{-\omega r_0/v}}{\sqrt{r_0^2-r^2}} + \frac{\omega}{v} \sqrt{r_0^2-r^2}\, e^{-\omega r_0/v} + \frac{\omega^2}{v^2} \int_r^{r_0} \sqrt{y^2-r^2}\, e^{-\omega y/v}\, dy \right\} J_1(kr\sin\vartheta)\, dr, \tag{6}$$

where $J_1(kr\sin\vartheta)$ is the Bessel function of the first order.

(6) is valid only for the case $kr_0 \ll 1$ ($\lambda \gg 2\pi r_0$), since retardation was not taken into account when calculating the field at the aperture. In this case

$$J_1(kr\sin\vartheta) \simeq \frac{1}{2} kr\sin\vartheta.$$

After substitution, (6) simplifies to

$$\mathbf{E}_2(M) = \frac{e_0 v \sin\vartheta}{\pi c^2} \frac{e^{-jkR_0}}{R_0} \left[1 - e^{-\omega r_0/v} \left(1 + \frac{\omega r_0}{v} \right) \right] \vec{\vartheta}_0. \tag{7}$$

Adding (7) and (1), we obtain the total radiation field

$$\mathbf{E}(\omega) = -\frac{e_0 v \sin\vartheta}{\pi c^2} \frac{e^{-ikR_0}}{R_0} e^{-\omega r_0/v} \left(1 + \frac{\omega r_0}{v} \right) \vec{\vartheta}_0. \tag{8}$$

It follows from (8) that the presence of the aperture leads to an attenuation of the dipole part of the transition radiation. Higher frequencies are attenuated more strongly. The total energy radiated by a point charge in the case of transition radiation from an ideally conducting plane is equal to ∞. The aperture in the

Fig. 2

Figure 2: Fig. 2

screen causes the radiation spectrum to be bounded on the high-frequency side; as a result, the total radiated energy is finite.

Fig. 2

The spectral density of the radiated energy is proportional to $E^2(\omega)$. The ratio of the spectral density of the energy radiated into one half-space to the spectral density of the transition-radiation energy is

$$f\left(\frac{\omega r_0}{v}\right) = \left[e^{-\omega r_0/v}\left(1 + \frac{\omega r_0}{v}\right)\right]^2.$$

This function is shown in Fig. 2.

The spectral density of the radiated energy is reduced by a factor of two in comparison with transition radiation at $\omega r_0/v = 1.07$. If $v/c = 0.1$ ($V_{\text{acc}} = 3$ kV) and the radius of the aperture is 0.5 mm, this corresponds to $\lambda = 3$ cm. This means that, by passing through this aperture a harmonic current wave of amplitude 10 mA, one can obtain a radiation power of 12 μW.

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Note: Figure translations are in progress. See original paper for figures.