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Abstract

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ON THE CONDITIONS FOR THE EXISTENCE OF A STIGMATIC IMAGE IN ELECTRON-OPTICAL SYSTEMS WITH A CURVILINEAR AXIS

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1. The existence of conjugate points ([1]) on the axis of an electron-optical system is equivalent to the existence of a real image. The real image will be stigmatic only for special types of electron-optical systems; in the general case the object point is imaged not as a point, but as a line segment. In the present work conditions are established that are sufficient for the existence of a stigmatic image in the paraxial region of a system with a curvilinear axis.

2. Let us take as the s-axis of a curvilinear system of coordinates s, p, q the principal trajectory of the beam (which is also the axis of the electron-optical system), and direct the p and q axes along the normal and binormal to the principal trajectory. Denote by k and $\varkappa$ the curvature and torsion of the principal trajectory; $\varphi(s,p,q)$, $\psi(s,p,q)$ the electrostatic and magnetic scalar potentials; $\Phi = \Phi(s)$, $\Psi = \Psi(s)$ the values of the potentials on the principal trajectory; $\varphi_i, \psi_i, \varphi_{ik}, \psi_{ik}$ $(i,k = 2,3)$ the values of the partial derivatives of the potentials on the s-axis (here the subscript 2 denotes differentiation with respect to p, the subscript 3 differentiation with respect to q); $-e, m$ the charge and mass of the electron. A prime will denote differentiation with respect to s.

On the basis of Poincaré' s theorem ([2]), the quantities $p = p(s)$, $q = q(s)$ for a paraxial trajectory may be represented in the form of series

$$\begin{aligned} p &= p^I + \lambda p^{\text{II}} + \lambda^2 p^{\text{III}} + \cdots \\ q &= q^I + \lambda q^{\text{II}} + \lambda^2 q^{\text{III}} + \cdots \end{aligned} \tag{1}$$

For a given electron-optical system, the series (1) will determine the electron trajectories for a definite value of the parameter λ. In order that these series converge more rapidly, when the parameter λ is introduced into the trajectory equations the special features of the class of electron-optical systems under study are taken into account. In considering systems that deviate only slightly from axially symmetric ones, λ is defined so that the system of equations of the first approximation is the system

$$\begin{aligned} L(p,q) &\equiv p'' + \frac{\Phi'}{2\Phi}p' - \alpha\Psi' q' + \frac{\Phi''}{4\Phi}p - \frac{1}{2}\alpha\Psi'' q = 0, \\ M(p,q) &\equiv q'' + \frac{\Phi'}{2\Phi}q' + \alpha\Psi' p' + \frac{1}{2}\alpha\Psi'' p + \frac{\Phi''}{4\Phi}q = 0. \end{aligned} \tag{2}$$

Then the subsequent approximations $p^{(n)}, q^{(n)}$ satisfy the system

$$L(p^{(n)}, q^{(n)}) = l(p^{(n-1)}, q^{(n-1)}), \qquad M(p^{(n)}, q^{(n)}) = m(p^{(n-1)}, q^{(n-1)}), \tag{3}$$

where

$$-l(p,q) = \frac{1}{d}\left(-2\varkappa q' + A_1 p + A_2 q\right),$$

$$-m(p,q) = \frac{1}{d}\left(2\varkappa p' + B_1 p + B_2 q\right); \tag{4}$$

$$\begin{aligned} A_1 &= \frac{1}{2\Phi}\left[\frac{3}{2}k\varphi_2 + \mu + 2\left(k^2 - \varkappa^2\right)\Phi\right] + \alpha\left(\psi_{23} + \frac{\varphi_2}{2\Phi}\psi_3 - \varkappa\Psi'\right) \\ A_2 &= \frac{1}{2\Phi}\left[2k\varphi_3 - 2\varkappa'\Phi - \varkappa\Phi' - \varphi_{23}\right] + \alpha\left(\nu + \frac{1}{2}k\psi_2 + \frac{\varphi_3}{2\Phi}\psi_3\right) \\ B_1 &= \frac{1}{2\Phi}\left[2\varkappa'\Phi + \varkappa\Phi' - \varphi_{23}\right] - \alpha\left(-\nu + \frac{1}{2}k\psi_2 + \frac{\varphi_2}{2\Phi}\psi_2\right) \\ B_2 &= -\frac{1}{2\Phi}\left[2\varkappa^2\Phi + \mu + \frac{1}{2}k\varphi_2\right] - \alpha\left(\psi_{23} + \frac{\varphi_3}{2\Phi}\psi_2 + \varkappa\Psi'\right) \end{aligned} \tag{5}$$

$$\alpha = \sqrt{\frac{e}{2m\Phi}}; \tag{6}$$

$$\mu = \frac{1}{2}\left(\varphi_{33} - \varphi_{22}\right); \tag{7}$$

$$\nu = \frac{1}{2}\left(\psi_{33} - \psi_{22}\right); \tag{8}$$

d denotes the greatest of the values $|2\varkappa|$, $|A_1|$, $|A_2|$, $|B_1|$, $|B_2|$ in the interval $[s_0, s_u]$.

Equations (2) may be used as equations of the first approximation also in the investigation of electron-optical systems whose deviations from axial symmetry cannot be regarded as small. However, if the electron-optical system Σ deviates only slightly from a system with a curvilinear axis Σ_0, then, in studying the focusing properties of Σ, it is convenient to take Σ_0 as the first approximation to Σ and, correspondingly, to determine the operators L, M, l, m.

3. Denote by r_1, r_2 the partial solutions of the equation

$$r'' + \left(\frac{3}{16}\frac{\Phi'^2}{\Phi^2} + \frac{e}{8m\Phi}\Psi'^2\right) r = 0, \tag{9}$$

defined by the initial conditions

$$r_1\big|_{s=s_0} = 0, \qquad r_1'\big|_{s=s_0} = 1,$$

$$r_2\big|_{s=s_0} = 1, \qquad r_2'\big|_{s=s_0} = 0. \tag{10}$$

The fundamental matrix of the system of the first approximation is written in the form

$$P \equiv \|p_{ik}\| = \Phi^{-1/4} \begin{pmatrix} r_2 \cos\theta & -r_2 \sin\theta & r_1 \cos\theta & -r_1 \sin\theta \\ r_2 \sin\theta & r_2 \cos\theta & r_1 \sin\theta & r_1 \cos\theta \end{pmatrix} \tag{11}$$

where

$$\theta = -\frac{1}{2}\int_{s_0}^{s_u} \alpha\Psi'\, ds. \tag{12}$$

The solutions of system (3) for the following approximations are found by the method of variation of arbitrary constants in the form of quadratures. We shall confine ourselves to taking into account the first two terms in the series (1); the conditions for the plane $s = s_u$,

normal to the principal trajectory, be a stigmatic image of the plane $s = s_0$, are written in the form

$$p_{13}^u + \sum_{i=1}^{4} C_i p_{1i}^u = 0, \qquad p_{23}^u + \sum_{i=1}^{4} C_i p_{1i}^u = 0,$$

$$p_{14}^u + \sum_{i=1}^{4} D_i p_{2i}^u = 0, \qquad p_{24}^u + \sum_{i=1}^{4} D_i p_{2i}^u = 0, \tag{13}$$

where it is denoted

$$p_{ik}^u = p_{ik}\big|_{s=s_u}; \tag{14}$$

$$C_i = \int_{s_0}^{s_u} \Phi\left[(-2\chi p_{23}' + A_1 p_{13} + A_2 p_{23})P_{3i} + (2\chi p_{13}' + B_1 p_{13} + B_2 p_{23})P_{4i}\right] ds,$$

$$D_i = \int_{s_0}^{s_u} \Phi\left[(-2\chi p_{24}' + A_1 p_{14} + A_2 p_{24})P_{3i} + (2\chi p_{14}' + B_1 p_{14} + B_2 p_{24})P_{4i}\right] ds \tag{15}$$

$$P_{31} = P_{41} = -\Phi^{-3/4} r_1 \cos\theta, \qquad P_{32} = -P_{42} = \Phi^{-3/4} r_1 \sin\theta,$$

$$P_{33} = P_{44} = \Phi^{-3/4} r_2 \cos\theta, \qquad P_{34} = -P_{43} = -\Phi^{-3/4} r_2 \sin\theta. \tag{16}$$

If, in particular, s_u is defined by the relation

$$r_1(s_u) = 0, \tag{17}$$

then the conditions (13) for a stigmatic image reduce to the relations:

$$\int_{s_0}^{s_u} r_1^2\left[A_1\cos^2\theta + (A_2 + B_1)\sin\theta\cos\theta + B_2\sin^2\theta + \alpha\chi\psi'\right] ds = 0,$$

$$\int_{s_0}^{s_u}\left[r_1^2\left(-A_2\sin^2\theta + (B_2 - A_1)\sin\theta\cos\theta + B_1\cos^2\theta\right) + 2\chi r_1\left(r_1' - \frac{\Phi'}{4\Phi}r_1\right)\right] ds = 0, \tag{18}$$

$$\int_{s_0}^{s_u}\left[r_1^2\left(A_2\cos^2\theta + (B_2 - A_1)\sin\theta\cos\theta - B_1\sin^2\theta\right) - 2\chi r_1\left(r_1' - \frac{\Phi'}{4\Phi}r_1\right)\right] ds = 0,$$

$$\int_{s_0}^{s_u} r_1^2\left[A_1\sin^2\theta - (A_2 + B_1)\sin\theta\cos\theta + B_2\cos^2\theta + \alpha\chi\psi'\right] ds = 0.$$

If further terms are taken into account in the series (1), then in the conditions (13) C_i, D_i $(i = 1, 2, 3, 4)$ must be replaced by the sums

$$C_i = \sum_s C_{is}, \qquad D_i = \sum_s D_{is},$$

where C_{is}, D_{is} are calculated by formulas (15) under the condition that, instead of p_{ik}, the corresponding approximations to the quantities p, q are substituted.

We note that, as a special case ($k = \chi = 0$), the known results ([3,4]) for systems with a rectilinear axis are obtained from the foregoing.

4. The results presented above can be successfully used in considering problems of a theoretical investigation of the action of a stigmator. For example, it is easy to see from (18) that, in an electrostatic electron microscope, the axial astigmatism arising because of the ellipticity of the objective apertures can be corrected by a stigmator whose field is determined by the formula

$$\varphi(s, p, q) = \Phi_1 + \Phi_2(s)(p^2 - q^2). \tag{19}$$

Here $\Phi_1 = \Phi\big|_{s=s_1}$, while s_1 is determined from the condition that, in the interval $[s_1, s_u]$ of action of the stigmator, the field of the objective is equal to zero. $\Phi_2(s)$ is a linear function of s; for constant Φ_2, the field (19) was used in a synchrotron [(5)].

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.