# Reports of the Academy of Sciences of the USSR

Corresponding Member of the Academy of Sciences of the USSR
**S. V. Vonsovskii** and **M. S. Svirskii

1958

SovietRxiv

## Abstract

## Full Text

**Reports of the Academy of Sciences of the USSR**

**1958. Vol. 122, No. 2**

## PHYSICS

Corresponding Member of the Academy of Sciences of the USSR **S. V. Vonsovskii** and **M. S. Svirskii**

# ON THE QUESTION OF THE ABSENCE OF SUPERCONDUCTIVITY IN FERROMAGNETICS

As is known, the property of superconductivity has not so far been found in ferromagnetic metals. In connection with this, V. L. Ginzburg ([1]) noted that this, possibly, does not yet indicate the absence of superconductivity in ferromagnetics, which, however, practically cannot be detected under ordinary conditions because of the masking action of the spontaneous magnetization. At the same time, of course, the question was not resolved as to whether the system of conduction electrons of a ferromagnetic possesses the property of superconductivity. It is therefore of interest to discuss this question from the standpoint of the new microscopic theory of superconductivity ([2]-[4]), taking into account the features of the energy spectrum of the conduction ($s$) electrons considered, for example, in the $(s-d)$-exchange model of ferromagnetic metals ([5]).

According to ([5]), owing to the $(s-d)$-exchange interaction, the number of $s$-electrons with spin projection $(-1/2)$ turns out to be different from their number with projection $(+1/2)$, and correspondingly there are also different maximum wave vectors $k_F$ (at $T=0$) for these electrons with left and right spin orientations. It is not difficult to show that

$$k_F(-1/2) - k_F(+1/2) \simeq \frac{2m}{\hbar^2 k_F} \frac{1}{[(1+\mu')^{1/3} + (1-\mu')^{1/3}]} \, \mu I_0, \tag{1}$$

where $m$ is the electron mass; $2\pi\hbar$ is Planck's constant; $k_F$ is the maximum wave vector for a nonferromagnetic metal with the same concentration of $s$-electrons as in the ferromagnetic under consideration;

$$\mu = \frac{n_d(-1/2) - n_d(+1/2)}{n_d(-1/2) + n_d(+1/2)}$$

(where $n_d(\sigma)$ is the number of $d$-electrons with spin orientation $\sigma = +1/2, -1/2$, so that for ferromagnetics $\mu \sim 1$);

$$\mu' = \frac{n_s(-1/2) - n_s(+1/2)}{n_s(-1/2) + n_s(+1/2)}$$

(where $n_s(\sigma)$ is the number of $s$-electrons with spin orientation $\sigma$); $I_0 = I(\mathbf{k}, \mathbf{k})$ is the integral of $(s-d)$-exchange, which we approximately regard as constant, i.e. independent of $\mathbf{k}$ ([6]). For definiteness we also take $I_0 > 0$ and, correspondingly, according to (1), we assume a predominance among the $s$-electrons of spins with left orientation.

Thus, in ferromagnetics in the normal state at $T = 0$ there are three regions in $\mathbf{k}$-space: in the first of them ($k < k_F(1/2)$—region I) there are equal (under the condition $I(\mathbf{k}, \mathbf{k}) = I_0 = \text{const}$) numbers of spins of the two possible orientations; in the second ($k_F(1/2) < k < k_F(-1/2)$—region II) there are only $s$-electrons with left spin orientation; and in the third ($k > k_F(-1/2)$—region III) there are only holes. This circumstance does not make it possible to apply, for clarifying the question of the possible existence in ferromagnetics of a superconducting state energetically more favorable than the normal one, the canonical transformation of the form ([2])

$$\alpha_{\mathbf{k}0} = u_{\mathbf{k}} a_{\mathbf{k},1/2} - v_{\mathbf{k}} a^*_{-\mathbf{k},-1/2}, \qquad \alpha_{\mathbf{k}1} = u_{\mathbf{k}} a_{-\mathbf{k},-1/2} + v_{\mathbf{k}} a^*_{\mathbf{k},1/2}; \tag{2}$$

$$u_{\mathbf{k}}^2 + v_{\mathbf{k}}^2 = 1, \tag{2'}$$

in which operators acting on the occupation numbers of particles with different spin orientations enter on an equal footing*. Indeed, the trivial solution of the compensation equation from work ([2]) has the form

$$u_k = \begin{cases} 1, & k > k_F; \\ 0, & k < k_F, \end{cases} \qquad v_k = \begin{cases} 0, & k > k_F; \\ 1, & k < k_F, \end{cases} \tag{3}$$

which corresponds to the normal state with electrons of the two possible spin orientations below the Fermi surface ($k < k_F$) and holes outside this surface, whereas the nontrivial solution corresponds to a superconducting state with $u_k \neq 0$ for $k < k_F$ (penetration of some holes below the Fermi surface) and $v_k \neq 0$ for $k > k_F$ (promotion of some electrons above the Fermi surface). According to the considerations set forth above, in ferromagnets one must distinguish in the normal state not two, but three regions in $k$-space, and correspondingly, instead of the two quantities $u_k$ and $v_k$, introduce four quantities: $u_k(-1/2)$, $v_k(-1/2)$, $u_k(1/2)$, $v_k(1/2)$, whose normal values will, evidently, have the following form:

$$u_k(-1/2) = \begin{cases} 1, & k > k_F(-1/2); \\ 0, & k < k_F(-1/2); \end{cases} \qquad v_k(-1/2) = \begin{cases} 0, & k > k_F(-1/2); \\ 1, & k < k_F(-1/2); \end{cases}$$

$$u_k(1/2) = \begin{cases} 1, & k > k_F(1/2); \\ 0, & k < k_F(1/2); \end{cases} \qquad v_k(1/2) = \begin{cases} 0, & k > k_F(1/2); \\ 1, & k < k_F(1/2). \end{cases} \tag{4}$$

If now we write the transformation (2), taking into account the difference in the distributions of particles with different spin projections, in the form

$$\alpha^*_{k0} = u_k(1/2)a_{k,1/2} - v_k(-1/2)a^*_{-k,-1/2},$$

$$\alpha^*_{k1} = u_k(-1/2)a_{-k,-1/2} + v_k(1/2)a^*_{k,1/2}, \tag{5}$$

then, to determine the 4 unknown quantities**, 4 equations will be required, one of which gives the compensation relation. The remaining 3 equations may be found from the requirement that for $\alpha_{k0}$ and $\alpha_{k1}$ the anticommutation relations of Fermi operators be preserved and that, in addition, the operator $\nu_{k0} = \alpha^*_{k0}\alpha_{k0}$ commute with $\nu_{k1} = \alpha^*_{k1}\alpha_{k1}$.** *It is not difficult, however, to see that the listed requirements are compatible with (4) and (1) only in the nonferromagnetic case, when $\mu I_0 = 0$ and, consequently, $k_F(-1/2) = k_F(1/2)$, region II is absent and (5) coincides with (2). Indeed, it is easy to verify that, for fulfillment of the relation**** $[\alpha^*_{k0}\alpha_{k'0}]_+ = \delta_{kk'}$, the following connection between $u_k(1/2)$ and $v_k(-1/2)$ is necessary:

$$u^2_k(1/2) + v^2_k(-1/2) = 1, \tag{6}$$

and, for fulfillment of the relation $[\alpha^*_{k1}\alpha_{k'1}]_+ = \delta_{kk'}$, it is necessary that

$$u^2_k(-1/2) + v^2_k(1/2) = 1. \tag{7}$$

Further, since $[\alpha^*_{k1}\alpha^*_{k0}]_+ = 0$, while $[\alpha_{k1}\alpha_{k0}] \neq 0$, for $\nu_{k0}$ and $\nu_{k1}$ to commute it is necessary that $[\alpha_{k1}\alpha_{k0}]_+ = 0$. Hence follows the relation

$$u_k(1/2)v_k(1/2) = u_k(-1/2)v_k(-1/2). \tag{8}$$

* The notation is the same as in ([2]).

** Formula (4) determines their values only for the normal state.

*** The last requirement is necessary so that the occupation numbers $\nu_{k0}$ and $\nu_{k1}$ can simultaneously have definite values, which is implied, for example, when writing the "vacuum" of the electron–phonon system without interaction in the form $c_\nu = \prod_k \delta(\nu_{k0})\delta(\nu_{k1})$ (see ([2])).

**** Here and below $[ab]_+ = ab + ba$ and $[ab]_- = ab - ba$.

But from (6), (7), and (8) it follows that

$$v_k(1/2) = v_k(-1/2), \qquad u_k(1/2) = u_k(-1/2), \tag{9}$$

which, together with (1) and (4), is possible only for $\mu I_0 = 0$, since for $\mu I_0 \neq 0$ $v_k(1/2)$ and $v_k(-1/2)$ differ from one another in region II, where, for example, in the normal state $v_k(1/2) = 0$, while $v_k(-1/2) = 1$. The fact that the presence in region II of left-hand ones is incompatible with the required permutation relation is already evident from the fact that, according to (4) and (5), in this region $\alpha_{k1}$ is simply equal to zero and therefore cannot satisfy the permutation relations for Fermi operators. Thus, in this region there is no "seed" for the phonon-induced interelectron interaction to be able to form a state with superpositions of right-hand electrons and left-hand holes, whose energy would be lower than in the normal state.

On the other hand, the very presence in region II of left-hand ones must play a negative role in establishing the property of superconductivity in the system of $s$-electrons. Pines ([7]) attempts to explain this by the fact that Coulomb repulsion almost balances the phonon-induced attraction between electrons with opposite momenta and spins, and that therefore a sufficiently small decrease (upon the emergence of region II) in the number of possible pairs $(k, 1/2; -k, -1/2)$ is enough for the Coulomb repulsion, which impedes superconductivity, to become predominant. This argument, based on the theory of Bardeen, Cooper, and Schrieffer ([3]), is apparently insufficient, since in the more exact theory of N. N. Bogolyubov et al. ([8]) the role of Coulomb repulsion is considerably weakened (owing to the presence in the denominator of $\ln(E_F/\hbar\tilde{\omega})$)* . A much greater decrease in the number of possible pairs is hindered by the accompanying increase in the kinetic energy in the formation of ones with parallel spins.

Therefore it seems to us that an important role is played here by the circumstance that the phonon-induced interaction gives an effective attraction only for electron transitions satisfying the following condition ([3]): $|\varepsilon_k - \varepsilon_{k+x}| < \hbar\omega_x$, where $x$ is the wave vector of the phonon. Conversely, under the condition $|\varepsilon_k - \varepsilon_{k+x}| \gg \hbar\omega_x$, the phonon-induced interaction leads to repulsion. Because region II is filled with "left-hand ones," the phonon-induced interaction must, in order to transfer a pair from region I to region III, impart to it at least an energy $\sim \mu I_0$; therefore such a transition can turn out to be energetically favorable only for $\mu I_0 < \hbar\tilde{\omega}$. Otherwise the inequality $|\varepsilon_k - \varepsilon_{k+x}| > \mu I_0 > \hbar\tilde{\omega}$ will hold, and, consequently, a repulsion will appear, making

* In this connection we draw attention to L. Brillouin' s note entitled "The Mirage of Superconductivity" ([9]), in which the influence of the Coulomb interaction of electrons on the phenomenon of superconductivity is denied altogether, on the grounds that this interaction, being internal, cannot affect the motion of the center of mass of the system of interacting particles. This consideration, however, seems unconvincing to us, since in the new microscopic theory of superconductivity the issue is the energy spectrum of a system of interacting electrons, on which the internal Coulomb forces act in such a way that they facilitate the destruction of the superconducting state by external influences.

See also our papers ([12]), in connection with which we take the opportunity to make the following remark. When that paper ([12]) was sent to press, we became acquainted with Pines' s work ([13]), where the question of the appearance of a superconducting state among elements of the periodic system is considered from the point of view of the theory set forth in ([3]). Pines ([13]) identifies the effective charge $Z_{\text{eff}}$ with the number of valence electrons $Z$. In doing so, he obtains significantly poorer agreement with experiment. Thus, for example, the elements Ga, Al, Re, Hg, Cd, Zn, Rn, and Os do not satisfy Pines' s proposed criterion of superconductivity. The agreement of this criterion with experiment is noticeably improved (above all for divalent elements) if, instead of $Z$, one substitutes $Z_{\text{eff}}$ from our paper ([12]). The latter circumstance apparently indicates that Matthias' s rule is an expression of essential properties of the superconducting state, for which an important role is played not by the number of valence electrons $Z$, but by a certain effective charge $Z_{\text{eff}}$.

energetically unfavorable for pairs to make the transition I→III. If the Coulomb repulsion is also taken into account, then it is clear that superconductivity is possible only under the condition $\mu I_0 \ll \hbar\tilde{\omega}$.

For a numerical estimate we shall take, in accordance with (7), $\hbar\tilde{\omega} \sim 10^{-14}$ erg, and put $I_0 \sim 10^{-13}$ erg for ferromagnets. For this value of $I_0$, the number of possible pairs decreases by only $\sim 1.5\%$ (see (1)), but this is already sufficient to violate the condition $\mu I_0 \ll \hbar\tilde{\omega}$, and therefore the superconducting state becomes energetically unfavorable.

Thus, we arrive at the conclusion that, from the standpoint of the new microtheory, the absence of superconductivity in ferromagnets is an intrinsic property of their electron system, caused by a relatively strong ($s$—$d$)-exchange interaction*. Therefore, in our opinion, superconductivity can be observed only in those ferromagnetic metals that possess a very weak ($s$—$d$)-exchange interaction ($\mu I_0 \ll \hbar\tilde{\omega}$). Consideration of such a case, carried out by us on the basis of the method of paper ([3]), shows that the behavior of the distribution function of pairs (and, correspondingly, of holes) in regions I and III remains essentially the same as in ([3]). Region II, however, turns out to be filled with left-hand units, since the transition of a unit from II to III is energetically unfavorable owing to the assumed effectiveness of the phonon-induced interaction only for pair transitions, while the transition II→I is energetically unfavorable because of the necessity of displacing a pair from region II. The presence of region II leads to a lowering of the transition temperature into the superconducting state, which, together with the action of the spontaneous magnetization, greatly complicates the detection of the property of superconductivity. Nevertheless, under special conditions (a sample with a ratio of thickness to transverse dimensions of the order of $10^{-4}$ ([11]) and very low temperatures), the detection of superconductivity in such ferromagnetic metals is nevertheless possible in principle.

Institute of Metal Physics
Ural Branch of the Academy of Sciences of the USSR

Received
12 VI 1958.

## CITED LITERATURE

[1] V. L. Ginzburg, ZhETF, **31**, 202 (1956).
[2] N. N. Bogolyubov, ZhETF, **34**, 58 (1958).
[3] J. Bardeen, L. N. Cooper, J. R. Schrieffer, Phys. Rev., **106**, 162 (1957).
[4] H. Fröhlich, Phys. Rev., **79**, 845 (1950).
[5] S. V. Vonsovskii, ZhETF, **16**, 987 (1946).
[6] S. V. Vonsovskii, E. A. Turov, ZhETF, **24**, 419 (1953).
[7] D. Pines, Phys. Rev., **109**, No. 2, 280 (1958).
[8] N. N. Bogolyubov, V. V. Tolmachev, D. V. Shirkov, *A New Method in the Theory of Superconductivity.* Publishing House of the Academy of Sciences of the USSR, 1958.
[9] L. Brillouin, J. Phys. et le Radium, **19**, No. 1, 112 (1958).
[10] Yu. P. Irkhin, *Fiz. met. i metalloved.*, **6**, No. 2, 3 (1958).
[11] G. F. Zharkov, ZhETF, **34**, 412 (1958).
[12] S. V. Vonsovskii, M. S. Svirskii, DAN, **120**, No. 2 (1958).
[13] D. Pines, Phys. Rev., **109**, 280 (1958).

---

* This same interaction leads in antiferromagnets (in which the numbers of *s*-electrons with opposite projections of the spins are equal to one another) to the formation of a region with a reduced density of states ([10]), which likewise impedes the transition of pairs, as does region II in the case of ferromagnets.

*Note: Figure translations are in progress. See original paper for figures.*