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**Abstract**

**Full Text**

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# On the Results of Determining the Electron Concentration of the Outer Region of the Ionosphere from Observations of the Radio Signals of the First Earth Satellite

*(Presented by Academician V. A. Kotel' nikov, 31 III 1958)*

**1.** We briefly describe the results of observations of the satellite' s radio signals, based on determining the times of its "radio rise" and "radio set." Data have been obtained on the distribution of the electron concentration $N$ of the ionosphere above its maximum $N_m$, and the considerations concerning the properties of interplanetary gas that follow from them.

The method of investigation is as follows. Suppose that a satellite emitting radio waves of frequency $f$ passes above the observation point, above the $N_m$ region of $F_2$, whose critical frequency is

$$(2\pi)^2 f_c^2 = \omega_c^2 = \frac{4\pi e^2}{m} N_m = 3.2 \cdot 10^9 N_m,$$

with $\omega > \omega_c$. Then it is expedient to consider two types of trajectories for propagation of the wave to the observation point, determined by the relations $\omega_\mathrm{e} : \omega < 1$ and $\omega_\mathrm{e} : \omega \geqslant 1$, where $\omega_\mathrm{e}$ is a certain limiting value of the frequency.

When $\omega_\mathrm{e} : \omega < 1$, the following picture is observed. If $\omega_c : \omega \ll 1$, then, obviously, the propagation is quasi-optical—the maximum horizontal range $r_m$ of reception of signals from the satellite is equal to $R_0\theta_0$, the distance of its optical visibility, and the wave trajectory coincides with the straight line tangent to the Earth at the observation point and connecting it with the emission point (0) in Fig. 1. As $\omega_c/\omega$ increases, the wave trajectory takes the form shown by curves (1) and (2) in Fig. 1. In some part of the lower ionosphere (region $z_0 \to z_m$) the wave trajectory (ray) is at first pressed toward the Earth, i.e., as the wave penetrates deeper into the ionosphere the angle $\theta$ between the normal to the wave front and the radius vector $R$ drawn from the center of the Earth to the corresponding point increases.

**Fig. 1**

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

At a certain height $(z_0 + z_\mathrm{e})$, lying below the height $(z_0 + z_m)$ of the maximum $N_m$, the value of $\theta$ becomes maximal and then decreases; this means that the ray has a point of inflection here. The corresponding values of $(z_0 + z_\mathrm{e})$ and $\theta_\mathrm{e}$ depend on the height $z_0$ of the beginning of the layer, its half-thickness $z_m$, and the ratio $\omega_c/\omega$. For given $z_0$ and $z_m$, the values of $\theta_\mathrm{e}$ and $z_\mathrm{e}$ increase with increasing $\omega_c/\omega$; at the same time $r_m$ also increases, and the wave trajectory has tangency at the observation point ($\theta_0 = \pi/2$). Finally, for the value $\omega = \omega_\mathrm{e}$, the angle $\theta_\mathrm{e} = \theta_{\mathrm{e}m} = \pi/2$, and the distance $r_m = r_{m\mathrm{e}} = R_0\theta_\mathrm{e}$ corresponds to the largest of its possible values for given

$\cdots z_0$ and $z_m$ is the extremal value of the maximum reception ranges $r_m$ (curve (e)). The height $(z_0 + z_e)$, at which $\theta_e = \pi/2$, is, as before, below the height of the layer maximum $(z_0 + z_m)$. For $\omega_e > \omega < \omega_c/\omega_e$, as is easy to understand, the maximum distance $r_m$ corresponds to those wave trajectories which also have, at the point of ray bending, the value $\theta_e = \pi/2$; however, at the reception point tangency is then no longer possible and $\theta_0 < \pi/2$ (curves (3), (4), (5) in Fig. 1). As $\omega$ decreases, the height $(z_0 + z_e)$ increases and reaches the height of the layer maximum $(z_0 + z_m)$ when $\omega_c : \omega = 1$ and $r_m = 0$—the ionosphere becomes opaque here to waves emitted from the satellite*.

Fig. 2

The method used in this work is based precisely on determining, from experimental data, the maximum ranges $r_m$ corresponding to the radio rise or radio set of the satellite, and comparing them with theoretically calculated values of $r_m$. In doing this, we restricted ourselves to considering data for which $\omega_e < \omega$, $\theta_e < \pi/2$, and the approximation of geometrical optics is suitable. In this case

$$r_m = R_0\vartheta_0 + \int_0^{z_c - z_0} \frac{dz}{\left(1 + \frac{z_0}{R_0} + \frac{z}{R_0}\right)\sqrt{n^2(z)\left(1 + \frac{z_0}{R_0} + \frac{z}{R_0}\right)^2 - 1}}, \qquad (1)$$

where all the notation is clear from Fig. 2 and $n(z)$ is the refractive index.

To calculate the integral it is necessary to choose the form of the function $n^2(z)$. In the region $z_0 \to z_m$, a good approximation for $n^2(z)$ is a parabola, which may be conventionally continued to $1.2z_m$. For the outer part of the ionosphere, where the electron concentration falls with height, one may choose an exponential law of decrease of $N(z)$. As a result, (1) is split into two integrals, in which, respectively,

$$n_1^2(z) = 1 - \frac{2\omega_c^2}{\omega^2}\frac{z}{z_m} + \frac{\omega_c^2}{\omega^2}\frac{z^2}{z_m^2}; \qquad n_2^2(z) = 1 - 0.96\frac{\omega_c^2}{\omega^2}e^{-x(z-1.2z_m)}. \tag{2}$$

Analysis of the radicand gives, for determining $z_e$, the quadratic equation

$$z^2 + (R_0 + z_0 - 3z_m)\frac{z}{2} + \left(\frac{z_m\omega^2}{2\omega_c^2} - R_0 - z_0\right)\frac{z_m}{2} = 0. \tag{3}$$

Its simultaneous solution with the equation

$$\left(1 + \frac{z_0}{R_0} + \frac{z_{em}}{R_0}\right)\left(1 - \frac{2\omega_e^2}{\omega_e^2}\frac{z_{em}}{z_m} + \frac{\omega_c^2}{\omega_e^2}\frac{z_{em}^2}{z_m^2}\right) = 0 \tag{4}$$

determines $\omega_c/\omega_e$ and $z_{em}$, where $\theta_e = \pi/2$.

Integral (1) is of elliptic type and was calculated by us numerically with the aid of the BESM machine for various values of $z_0$, $z_m$, $z_c$, and $\omega_c/\omega$**.

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* It should be noted that in the literature, when describing the character of wave trajectories, it is usually considered that the change in the sign of the curvature of the wave trajectory occurs at all frequencies at the height $(z_0 + z_m)$ of the maximum electron concentration. At the same time, this is correct only for a "flat" Earth. When the sphericity of the Earth is taken into account, the character of the trajectory changes; moreover, for calculating the frequency $\omega_e$, usually called the maximum usable communication frequency, it is necessary to use formulas different from the usual ones (see below).

** The analysis of the integral, its programming for the BESM, and the computations were carried out jointly with Z. A. Malina and E. P. Nesterova.

During continuous observation of the satellite' s radio signals, the times of their appearance and disappearance were recorded. From ballistic data for these moments, the altitude of the satellite, the coordinates of the point on the Earth over which it was passing, and its horizontal range $r_m$ were determined. After this, with the aid of synoptic maps—constructed specially for these investigations from ionospheric-station data—of isolines of the critical frequencies $f_c$, of the maps of the heights of the beginning $z_0$ and maximum $z_m$ of the $F_2$ region of the ionosphere, which played the principal role in these investigations, the values of $f_c$, $z_0$, and $z_m$ in the lower part of the ionosphere were determined for each selected observation time. Then, from comparison of the experimental values of $r_m$ with the theoretical course of the dependence of $r_m$ on $\varkappa$ for the selected $z_0$, $z_m$, and $\omega_c$, the value of $\varkappa$ was chosen which determined, for the corresponding

Fig. 3

Figure 3: Fig. 3

observation time, the effective rate of decrease of electron concentration above the ionospheric maximum $N_m$.

Fig. 3

2. The results of reception of the satellite's radio signals on October 5, 6, and 7 at 6 points were considered. Cases were selected in which a clear radio rise or radio set of the satellite was observed at a frequency of 40 Mc/s. The distribution curves of the ionospheric parameters characterizing these selected points give the following most frequently occurring values for this period: $z_0 \simeq 200$ km, $z_m \simeq 120$ km, $N_m \simeq 1.8 \cdot 10^6$, $z_c \simeq$ 600–650 km. The moments of local time near the satellite and the latitudes of these points varied within the limits $t \simeq$ 07 hr 40 min–09 hr 40 min and $\varphi \simeq (20 \div 45)^\circ$ N lat.

As a result of processing all the experimental data, the distribution curve of the values of $\varkappa$, shown in Fig. 3, was obtained, with the most frequently occurring value $\varkappa \sim 3.5 \cdot 10^{-3}$ km$^{-1}$ in the altitude range $z \sim 320 \div 600$ km.

At $z \gtrsim$ 500–600 km (the so-called exosphere), the gas temperature should not be lower than the temperature at lower levels. The state of ionization here is close to quasi-stationary (see below $\tau_e$); therefore, assuming that at $z > 600$ km the same rate of decrease of the electron concentration is preserved as at $z \sim$ 320–600 km, we apparently must obtain values of $N(z)$ that are underestimated in comparison with the true values. The corresponding values of $N(z)$ for $\varkappa = 3.5 \cdot 10^{-3}$ km$^{-1}$ are given in Table 1 and in Fig. 4.**

**Table 1**

| $z$, km | 200 | 320 | 400 | 1100 | 1760 | 2400 | 3050 |
|---|---|---|---|---|---|---|---|
| $N$, el/cm$^3$ | $10^5$ | $1.8 \cdot 10^6$ | $1.4 \cdot 10^6$ | $10^5$ | $10^4$ | $10^3$ | $10^2$ |
| $n$, cm$^{-3}$ | $5 \cdot 10^9$ | $2 \cdot 10^8$ | $10^8$ | $10^5$ | $10^2$ | $< 1$ | – |
| $n \simeq \frac{\tau_m}{\tau_e} N$ | – | – | $5 \cdot 10^8$ | $2 \cdot 10^5$ | $2 \cdot 10^3$ | 20 | $< 1$ |

* The ionospheric maps were constructed by E. E. Malishevskaya.

** *Note added in proof.* In Fig. 4 two values of $N$ measured during the launch of a high-altitude rocket are plotted (as points) (see the newspaper *Pravda*, March 27, 1958). It should also be noted that a recently published work (5) showed

Fig. 4

Figure 4: Fig. 4

that the ratio of the total number of electrons above the layer maximum to the number of electrons in the lower half of the layer, i.e.

$$\int_{z_0+z_m}^{\infty} N\,dz : \int_{z_0}^{z_0+z_m} N\,dz \simeq 3$$

changes little. This is also in good agreement with our data, since for $\varkappa \sim 3.5 \cdot 10^{-3}$ this ratio is of the order of 3.6.

In the 2nd line of Table 1 are also given values of the density $n$ of neutral particles recommended in various works ([1]) up to an altitude of 400 km, and for greater altitudes calculated on the assumption that the barometric formula is applicable and that here $T \sim 2000^{\circ}$, while the main gaseous component is atomic oxygen, so that the reduced height is $H \sim 100$ km. In the 3rd line are given the values of $n$ calculated below.

Let us now analyze the data of Table 1.

**Fig. 4**

First, it is seen that at $z \sim (2000–3000)$ km the electron concentration, under the assumptions made above, varies within the limits $10^3$–$10^2$ el/cm$^3$, and, naturally, in this region there is the same number of positive ions. But the considerable mean free paths, reaching here many thousands of kilometers when the particles move away from the Earth, the conditions of illumination of the gas by the Sun, the long lifetimes of the electron, the free influx of particles from outside, etc., indicate that here the gaseous medium can hardly be regarded as the atmosphere of the Earth. It may therefore be concluded that at these heights the atmosphere is apparently in contact with interplanetary gas*.

Second, let us estimate from the data on electron concentration the distribution of neutral particles $n$ for $z > 400$ km. At present one may adopt the following values of the flux of incident solar radiation $S$, ionizing the $F_2$ region of the ionosphere, and of the electron recombination coefficient $\alpha$ at various heights ([3,4]):

| $z_m$, km | $S_m$, erg/cm$^2 \cdot$ sec | $\alpha_m$, cm$^3$/sec |
|---|---|---|
| $\sim 320$ | $\sim 0.2$ | $\sim 10^{-10}$ |
| $\sim 400$ | $\sim 0.3$ | $\sim 10^{-11}$ |
| $z \geq 1000$ | $S_\infty \sim 0.6$ | $\alpha_\infty \sim 10^{-12}$ |

Therefore, taking into account that the principal microprocesses in this region of the atmosphere are photorecombination and photoionization, we obtain that the electron lifetime is $\tau_e = (\alpha N)^{-1}$ sec and the time between separate acts of ionization is $\tau_n = (\sigma S/\varepsilon_i)^{-1}$ sec. We take the values: $\sigma_{\text{O1}} \sim 2 \cdot 10^{-18}$ cm$^2$, and $\varepsilon_{i\text{O1}} = 13.5$ eV

| $z$, km | 320 | 400 | 1150 | 1850 | 2450 |
|---|---|---|---|---|---|
| $\tau_n$ | $5 \cdot 10^7$ | $3 \cdot 10^7$ | $2 \cdot 10^7$ | $2 \cdot 10^7$ | $2 \cdot 10^7$ |
| $\tau_e$ | $5 \cdot 10^3$ | $7 \cdot 10^4$ | $10^7$ | $10^8$ | $10^9$ |

Since under quasi-stationary conditions $n/N \sim \tau_n/\tau_e$, the result is the densities of neutral particles given in the 3rd line of Table 1. These values of $n$, as we see, exceed by a factor of 5-10 and more the data of the 2nd line of Table 1. It may therefore be assumed that at $z \sim 320$–$400$ km the values of $n$ are also greater than those given in the literature, while at $z \sim 2000$–$3000$ km $n$ is of the order of several units per 1 cm$^3$.

The conclusions drawn are of fundamental importance for the physics of the outer ionosphere, which emphasizes the need for their further verification by other methods.

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* We note here that such a value of the density of interplanetary gas has been obtained from observations of the polarization of zodiacal light ([2]).

*Note: Figure translations are in progress. See original paper for figures.*