Reports of the Academy of Sciences of the USSR

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1958

SovietRxiv

Abstract

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Reports of the Academy of Sciences of the USSR
1958, Volume 118, No. 5

ASTRONOMY

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FORMATION OF THE GAS TAIL OF COMETS

(Presented by Academician V. G. Fesenkov, 12 IX 1957)

As shown by the author ([1,4]), if the equations of motion of the molecules forming the atmosphere of a comet are specified:

$$x = f_1(\alpha, \varphi, t), \qquad y = f_2(\alpha, \varphi, t), \qquad z = f_3(\alpha, \varphi, t), \tag{1}$$

where x, y, z are cometocentric rectangular coordinates; α, φ determine the direction of ejection; t is time (with $x = y = z = 0$, $t = 0$), then the spatial density can be calculated from the formula

$$\rho = \frac{n \cos\alpha}{J}, \qquad \text{where } J = \frac{D(x, y, z)}{D(\alpha, \varphi, t)}. \tag{2}$$

The initial velocity v_0 and the acceleration of the repulsive force g, which for molecules of one type may be regarded as constant for small changes in the heliocentric distance of the comet, enter into equations (1) as parameters. In formula (2), n is the emission coefficient. Generally speaking, $n = n(\alpha, t)$, but in certain fairly common cases $n = \text{const}$. The dependence between n and α disappears, for example, in isotropic emission occurring during photodissociation of parent molecules. Constancy of n in time can also often be assumed for small intervals t, during which the molecules are located in the head of the comet.

Knowing ρ, one can calculate the visible density

$$N(x, y) = \int_{z_1}^{z_2} \rho \, dz \quad \text{or} \quad N(x, y) = 2 \int_{t_1}^{t_2} \rho_1 \, dt \left(\rho_1 = \rho \frac{dz}{dt} \right). \tag{3}$$

In formula (3), z_1 and z_2, or t_1 and t_2, are the smallest and largest values of z or t for given x, y in the case of a stationary distribution of visible density. During the formation of the head (a nonstationary distribution), t_2 must be replaced by the variable t, which determines the time elapsed from the beginning of emission to the moment of observation. In deriving formulas (3), we take the direction of the Oz axis to coincide with the direction of the line of sight.

Let us consider the process of formation of the head for the following special case. The molecules forming the head of the comet are produced as a result of photodissociation of parent molecules emitted from the solid nucleus. The velocity acquired in this case by the molecules is equal to v_0. Emission over a certain interval of time is constant. To simplify the calculation of isophotes, we regard the nucleus (the center of inertia of the comet) as motionless relative to the Sun. Under the action of an external force field, the molecules acquire a constant acceleration g. The equations of motion under these conditions are

$$x = v_0 t \sin\alpha - \frac{1}{2} g t^2; \qquad y = v_0 t \cos\alpha \cos\varphi; \qquad z = v_0 t \cos\alpha \sin\varphi \tag{4}$$

or

$$\frac{1}{4} g^2 t^4 - (v_0^2 - gx) t^2 + x^2 + y^2 + z^2 = 0. \tag{5}$$

Spatial density according to formula (2)

$$\rho = \frac{ng^2}{2v_0} \left[\frac{1}{-(A\sqrt{C^2 - g^2 z^2})\sqrt{C^2 - g^2 z^2}} + \frac{1}{(A + \sqrt{C^2 - g^2 z^2})\sqrt{C^2 - g^2 z^2}} \right]; \tag{6}$$

n is the emission coefficient; $A = v_0^2 - gx$; $C^2 = v_0^4 - 2v_0^2 gx - g^2 y^2$.

Taking t as the variable of integration, we find the visible density

$$N(x, y) = \frac{2n}{v_0} \int_{t_1}^{t} \frac{dt}{t\sqrt{-\frac{1}{4} g^2 t^4 + At^2 - (x^2 + y^2)}}$$

$$= \frac{2n}{v_0 \sqrt{x^2 + y^2}} \operatorname{arc\,tg} \frac{t_2}{t_1} \sqrt{\frac{t^2 - t_1^2}{t_2^2 - t^2}}. \tag{7}$$

Here the upper limit t is the time elapsed since the beginning of the constant emission, varying in the interval $t_1 \leq t \leq t_2$, where $t_1 = \sqrt{2(A - C)}/g$, $t_2 = \sqrt{2(A + C)}/g$; t_1 and t_2 are determined from equation (5) at $z = 0$ and are the roots of the polynomial under the radical in integral (7).

$$\text{For } t = t_2 \qquad N(x, y) = \frac{\pi n}{v_0 \sqrt{x^2 + y^2}}. \tag{8}$$

The isophotes calculated from formula (8) are circles whose center coincides with the photometric center of the comet. As calculations show, the deviations of the nonstationary distribution (7) from the stationary one (8) are small and do not exceed the errors of modern observations.

The envelope surface for the family (5) is a paraboloid of revolution, whose axis of symmetry coincides with the Ox axis, and whose vertex is at the point $(v_0^2/2g, 0, 0)$. To reach this point the molecule requires a time v_0/g. A feature of the spatial-density distribution (6) is that in the region $x > 0$ (in the direction toward the Sun) only molecules for which $x \leq v_0^2/2g$, and $t \leq 2v_0/g$, can be observed, whereas for $x < 0$ any values of t and x are theoretically admissible. It may be noted that ρ is a function symmetric with respect to the Ox axis—the radius vector of the comet.

Let us suppose that at some moment $t = T$ photoionization occurs of the molecules forming the head of the comet. In this case g may change discontinuously, acquiring a new value g_+, and the radiative capacity of the molecule may change; this can be allowed for by assuming a discontinuous change of the emission coefficient, so that

$$\text{for } t > T \qquad g_+ \neq g, \qquad n^+ \neq n. \tag{A}$$

The change in g will affect equation (4), but only for x. Namely, for $t > T$, $\dot{x} = v(T) + g_+(T - t)$, where $v(T) = v_0 \sin\alpha - gT$. The equations of motion of the ionized molecules will be:

$$\begin{aligned} x &= v_0 t \sin\alpha - \frac{1}{2}(g_+ - g)(t - T)^2 - \frac{1}{2}gt^2; \\ y &= v_0 t \cos\alpha \cos\varphi; \qquad z = v_0 t \cos\alpha \sin\varphi, \end{aligned} \tag{9}$$

or, after eliminating α and φ:

$$\left[x + \frac{1}{2}(g_+ - g)(t - T)^2 + \frac{1}{2}gt^2\right]^2 + y^2 + z^2 = v_0^2 t^2. \tag{10}$$

Under all these conditions, the visible density must be calculated from the formula

$$N(x, y) = 2\int_{t_1}^{T} \rho_1 \, dt + 2\int_{T}^{t_3} \rho_1^+ \, dt, \tag{11}$$

where t_3 is the time elapsed from the beginning of emission to the moment of observation, $t_1 < T < t_3$. The integrand ρ_1^+ is calculated in the usual way, but taking into account equations (9) and $n^+ \neq n$. The first integral is obtained from formula (7) for $T < t_2$. Carrying out the necessary calculations, we find

$$N(x,y) = \frac{2n}{v_0\sqrt{x^2+y^2}} \operatorname{arctg} \frac{t_2}{t_1}\sqrt{\frac{T^2-t_1^2}{t_2^2-T^2}} + \frac{2n^+}{v_0}\int_T^{t_3} \frac{dt}{t\sqrt{P(t)}}, \tag{12}$$

where

$$P(t) = at^4 + bt^3 + ct^2 + dt + e, \qquad a = -{}^1\!/_4 g_+^2, \qquad b = g_+(g_+ - g)T,$$

$$c = v_0^2 - g_+x + (g_+ - g)({}^3\!/_2 g_+ - g)T^2, \qquad d = (g_+ - g)\left[2x + (g_+ - g)T^2\right]T,$$

$$e = -{}^1\!/_4 (g_+ - g)^2T^4 - (g_+ - g)T^2x - (x^2 + y^2).$$

When t_3 is a root of $P(t)$, the second integral in (12) becomes improper, but convergent throughout the entire region of interest to us. Indeed, the equations $P(t) = 0$ and $P_t'(t) = 0$ have common roots only on the envelope, while inside it they have no joint solution, which proves the convergence of this integral.

In the case where t_3 is a root of the polynomial $P(t)$, the latter may be represented as

$$P(t) = (t_3 - t)(At^3 + Bt^2 + Ct + D), \tag{13}$$

where $A = -a,\ B = -(b + at_3),\ C = -(c + bt_3 + at_3^2),\ D = (d + ct_3 + bt_3^2 + at_3^3)$.

Using (13) and applying the mean-value theorem, we obtain the final expression for the visible density

$$N(x,y) = \frac{2n}{v_0\sqrt{x^2+y^2}} \operatorname{arctg} \frac{t_2}{t_1}\sqrt{\frac{T^2-t_1^2}{t_2^2-T^2}} +$$

$$+ \frac{2n^+}{v_0\sqrt{t_3(A\tau^3 + B\tau^2 + C\tau + D)}} \ln \frac{\sqrt{t_3} + \sqrt{t_3 - T}}{\sqrt{t_3} - \sqrt{t_3 - T}}, \tag{14}$$

with $T < \tau < t_3$.

Formula (14) is suitable in the case of a stationary distribution of visible density. In the region of a nonstationary distribution, t_3 is not a root of $P(t)$; consequently, the integrand of the second integral of formula (12) is continuous over the entire interval (T, t_3), including its boundaries. Then the integral may be estimated by any method accepted for proper integrals. It is evident that by a suitable choice of the parameters g_+ and n_+, in a certain region $N(x,y)$, calculated by formula (14), can be made greater than $N(x, y)$ according to formula (7). In this case the circular form of the isophotes in the given region will disappear, and an elongated tail will be obtained with maximum brightness on the Ox axis. The circular form of the isophotes will be preserved only in that region in which ionized molecules are not observed.

An example of such a deformation of the isophotes is given by B. A. Vorontsov-Velyaminov ([3]). He studied, with a Crosso self-recording microphotometer, photographs of the comet of 1942 obtained between 3 II and 31 III 1943. The isophotes were circular on the side facing the Sun and sharply elongated in the opposite direction, with the maximum visible density practically on the radius vector. Where the isophotes were circles, there was observed—

the law $N \sim r^{-1}$. The dimensions of this comet' s head exceeded $2 \cdot 10^6$ km. During this period the integral brightness and the heliocentric distance of the comet changed so little that it proved possible to assume constancy of the emission. Under this assumption, if from observations we knew x_0—the abscissa of the vertex of the enveloping paraboloid—and t_0—the time at which the stationary distribution sets in in the region $x \gg 0$, then, solving the system $x_0 = v_0^2/2g$ and $t_0 = 2v_0/g$, it would be possible to find the parameters v_0 and g. The work of B. A. Vorontsov-Vel' yaminov proves irrefutably that, with modern observational methods, it is impossible to determine with confidence the quantities x_0 and t_0. The author has made an attempt to compute all the physical parameters of interest to us by selection. The following values were obtained: $v_0 = 4 \cdot 10^5$ cm/sec; $g = 0.2$; $g_+ = 1.0$; $T = 45 \cdot 10^5$ sec., $n^+/n = 20$. Of these quantities one can only say that they do not contradict the observations and give a form of the isophotes similar to that given in the work of B. A. Vorontsov-Vel' yaminov.

Generally speaking, if we have at our disposal an isophote with equation

$$N(x,\ y,\ v_0,\ g,\ g_+,\ T,\ n^+/n) = C_1,$$

then, with a successful selection of parameter values at 5 suitable points (x_i, y_i), the computed and observed values of N should coincide. If there is a discrepancy between N_c and N_0, a correction of the adopted parameter values will be required, which apparently can be done. The basic condition is that the molecules forming the comet' s atmosphere be of the same type. This is more often encountered in faint comets, such as, for example, the comet of 1942.

Attention should also be paid to the fact that, in our work, for simplification we took g and g_+ to be constant. With significant changes in the heliocentric distances during the time of observation, one must remember that g and g_+ will be functions of t, and make the corresponding changes in the equations of motion. Unfortunately, at present there is not enough observational material for such investigations. Work (3) is the only example of how one should approach the reduction of comet observations in this direction. Hence one must draw one further conclusion. Determinations of v_0 and g, carried out by many authors up to the present time from the form of the tails and head of a comet, cannot be regarded as reliable. Into the theory of the question, beginning with the time of F. A. Bredikhin and up to the present moment, too many unfounded intuitive premises have been introduced, and conclusions have been drawn on the basis of chance (not specially arranged) observations. These shortcomings must be gradually eliminated in constructing a theory of cometary forms.

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Received
12 VI 1957

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Note: Figure translations are in progress. See original paper for figures.