# THEORY OF ELASTICITY

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**Abstract**

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## THEORY OF ELASTICITY

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# ON THE STABILITY OF CYLINDRICAL SHELLS UNDER DYNAMIC LOADING

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Let a gently sloping circular cylindrical panel, having a certain initial deflection, be subjected to rapid loading by compressive forces along the generatrix. We shall investigate the phenomenon of panel snap-through in time. We take the system of differential equations describing large deflections of a thin shell in the form

$$\frac{D}{h}\nabla^2\nabla^2(w_{\mathrm{p}}-w_{\mathrm{n}}) = w_{\mathrm{p},xx}\Phi_{,yy}+w_{\mathrm{p},yy}\Phi_{,xx}-2w_{\mathrm{p},xy}\Phi_{,xy}+\frac{1}{R}\Phi_{,xx}-\frac{\gamma h}{g}w_{\mathrm{p},tt}, \quad (1)$$

$$\frac{1}{E}\nabla^2\nabla^2\Phi = (w_{\mathrm{p},xy})^2 - w_{\mathrm{p},xx}w_{\mathrm{p},yy} - (w_{\mathrm{n},xy})^2 + w_{\mathrm{n},xx}w_{\mathrm{n},yy} - \frac{1}{R}w_{,xx}. \quad (2)$$

Here $w_{\mathrm{p}}(x,y,t)$ is the total deflection; $w_{\mathrm{n}}(x,y)$ is the initial deflection; $\Phi$ is the stress function in the middle surface; $R$ is the radius of curvature of the middle surface; $h$ is the shell thickness; $D$ is the cylindrical stiffness; $\gamma$ is the specific weight of the material; $x, y$ are coordinates measured along the generatrix and along the arc; $t$ is time. The indices after the comma denote differentiation with respect to the corresponding variable; $\nabla^2$ is the Laplace operator. In addition to the ordinary equations (1), an inertial term corresponding to the displacement $w$ has been introduced here.

We shall assume the edges of the panel to be simply supported. We approximate the deformed surface by means of the expression $w_{\mathrm{p}} = f_{\mathrm{p}}\sin(\pi x/a)\sin(\pi y/b)$, where $a, b$ are the length and width of the panel. For the initial deflection we take an analogous expression: $w_{\mathrm{n}} = f_{\mathrm{n}}\sin(\pi x/a)\sin(\pi y/b)$. Substituting these expressions into the right-hand side of equation (2) and, after integration, we find the function $\Phi$. In this case the expression for $\Phi$ will contain the term $-py^2/2$, where $p$ is the average intensity of the compressive forces. We then apply the Bubnov-Galerkin method to equation (1). We thus arrive at a nonlinear ordinary differential equation of the second order for the deflection amplitude $f_{\mathrm{p}}$.

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

Consider the case in which the curved edges of the panel approach one another with a prescribed velocity $c$, as occurs in many testing machines. We assume $c \ll V$, where $V$ is the velocity of sound propagation in the shell material; this assumption makes it possible to neglect the inertial terms corresponding to the displacements $u$ and $v$ in the middle surface. We determine the approach of the edges by the formula

$$e = -\frac{1}{b}\int_0^a \int_0^b \left[\frac{1}{E}\left(\Phi_{,yy} - \mu\Phi_{,xx}\right) - \frac{1}{2}(w_{\text{p},x})^2 + \frac{1}{2}(w_{\text{n},x})^2\right] dx\, dy; \quad (3)$$

$\mu$ is Poisson' s ratio. We take $e = ct$. In the case of a square panel ($a = b$), the equation for the deflection arrow takes the form

$$\frac{d^2\zeta_\text{p}}{dt^{*2}} - S\left\{\left[t^* - \frac{\pi^2}{4p_\text{B}^*}\left(\zeta_\text{p}^2 - \zeta_\text{H}^2\right) - \frac{k(1-\mu)}{\pi^2 p_\text{B}^*}\left(\zeta_\text{p} - \zeta_\text{H}\right)\right]\zeta_\text{p} + \frac{2k}{3\pi^2 p_\text{B}^*}\left(5\zeta_\text{p}^2 - 4\zeta_\text{p}\zeta_\text{H} - \zeta_\text{H}^2\right) - \left(\zeta_\text{p} - \zeta_\text{H}\right)\right\} = 0. \quad (4)$$

Here the following dimensionless parameters have been introduced:

$$k = \frac{b^2}{Rh}, \qquad \zeta_\text{p} = \frac{f_\text{P}}{h}, \qquad \zeta_\text{H} = \frac{f_\text{H}}{h}, \qquad S = \pi^2\left(p_\text{B}^*\right)^3\left(\frac{V}{c}\right)^2\left(\frac{h}{a}\right)^6,$$

$$t^* = \frac{ctb}{h^2 p_\text{B}^*}, \qquad p_\text{B}^* = \frac{p_\text{B} b^2}{Eh^2};$$

by $p_\text{B}$ is meant the upper critical stress for a panel having no initial deflection ([1]).

Equation (4) was integrated by a numerical method, and also with the aid of an electric analog computer of the MPT-9 type, for various values of $\zeta_\text{H}$ and $S$. The initial conditions adopted were: $\zeta_\text{p} = \zeta_\text{H}$ and $\zeta_{\text{p},t^*} = 0$ at $t = 0$.

**Fig. 1**

**Fig. 2**

In Fig. 1 are presented the results of computations relating to the limiting case of a **flat** plate ($k = 0$). Along the abscissa axis is plotted the time parameter

$t^*$, proportional to $e$, and along the ordinate axis—the dimensionless deflection arrow $\zeta_p$ and the ratio of the magnitude of the compressive force to the critical value $p/p_{cr}$. It was assumed that $\zeta_H = 0.001$ and $S = 100$. If the deflection arrow is determined by means of the static relations, then on the graph we obtain the segment $OB$, and then the dashed line $BLF$. Under dynamic loading the deflections increase on the segment $BM$ (solid line) very slowly, lagging behind the static ones. Subsequently the deflections increase rapidly (segment $MN$), after which steady nonlinear oscillations arise. The load increases on the segment $AK$ to a magnitude considerably exceeding the critical one. Similar results were obtained by Hoff et al. ([2]) for rods. The equation of type (1) was in that case linear.

In Fig. 2 are given results relating to a **cylindrical panel** with curvature parameter $k = 12$; the values $\zeta_H = 0.001$ and $S = 100$ are left as before. The static solution of the problem is represented here by the graph $\zeta_p(t^*)$, consisting of the segment $OB$ and the dashed line $BLF$. In the postcritical stage the quantity $t^*$ (or $e$) at first decreases (segment $BL$); this means that the loaded edges move somewhat apart. The load falls to the lower

of the critical value $p_n$, corresponding to the point $P$. Under dynamic application, the load increases, as for the plate, reaching 165% of the upper critical value. Then a sharp drop in $p$ occurs, and the value of $p$ becomes negative, which corresponds to tension of the panel. The graph in Fig. 2 describes the process of snap-through of the shell as it proceeds in time. Under real conditions the subsequent oscillations will quickly die out. The effect of dynamic loading is intensified as $\zeta_n$ or $S$ decreases. A family of curves $p(t^*)$ makes it possible, for a given $S$, to determine the influence of initial imperfections on the load-carrying capacity of the shell.

Similar results were obtained by V. L. Agamirov and the author for the case in which a closed circular cylindrical shell is subjected to axial compression or to all-round pressure. In the latter case it was assumed that the law of variation of the load with time was prescribed, and the deflected surface was approximated by means of expression (3)

$$\Big[\omega_p = (f_n + f)(\sin\alpha x \sin\beta y + \psi \sin^2\alpha x + \varphi). \tag{5}$$

Here $\alpha = \pi/L$; $\beta = ny/R$; $L$ is the length of the shell; $f$ is the amplitude of the additional deflection. The parameter $\varphi$ is determined from the condition of closedness. After two Bubnov–Galerkin equations were formulated to determine the parameters $f$ and $\psi$, the number $n$ of waves in the circumferential direction was additionally varied.

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3. A. S. Vol' mir, *DAN*, 113, No. 2 (1957).

*Note: Figure translations are in progress. See original paper for figures.*