ON THE COMPOSITION AND PARTITIONS OF NONPRIMITIVE CIRCUITS

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Abstract

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ON THE COMPOSITION AND PARTITIONS OF NONPRIMITIVE CIRCUITS

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In the present note we consider circuits formed by the composition of nonprimitive circuits, as well as by the partition of nonprimitive circuits into subcircuits. The definitions and notation correspond to those adopted in ([1]).

1. Let $X = (x^1, \dots, x^p)$ and $Y = (y^1, \dots, y^q)$ be sets of quantities x^i $(i = 1, 2, \dots, p)$, y^j $(j = 1, 2, \dots, q)$, taking the values either 0 or 1, and let $\widetilde{X} = (\tilde{x}^0, \dots, \tilde{x}^{2p-1})$, $\widetilde{Y} = (\tilde{y}^0, \dots, \tilde{y}^{2q-1})$ be the corresponding simple vectors. We shall call the set $Z = (z^1, \dots, z^{p+q})$ such that $z^1 = x^1, \dots, z^p = x^p$, $z^{p+1} = y^1, \dots, z^{p+q} = y^q$ the **union** of the sets X, Y. In this case the set Z will correspond to the simple vector $\widetilde{Z} = (\tilde{z}^0, \dots, \tilde{z}^{2p+q} - 1)$, which is the **union** of the simple vectors $\widetilde{X}, \widetilde{Y}$. We shall use the notation $Z = X \times Y$, $\widetilde{Z} = \widetilde{X} \times \widetilde{Y}$. Between the coordinates of the simple vectors $\widetilde{X}, \widetilde{Y}$ and $\widetilde{Z}$ the following relations hold:

$$\tilde{z}_{\gamma_{p+q},\dots,\gamma_{p+1},\gamma_p,\dots,\gamma_1} = \tilde{x}_{\gamma_p,\dots,\gamma_1} \tilde{y}_{\gamma_{p+q},\dots,\gamma_{p+1}}; \tag{1}$$

$$\tilde{x}_{\gamma_p,\dots,\gamma_1} = \bigvee_{\gamma_{p+q},\dots,\gamma_{p+1}} \tilde{z}_{\gamma_{p+q},\dots,\gamma_{p+1},\gamma_p,\dots,\gamma_1};$$

$$\tilde{y}_{\gamma_{p+q},\dots,\gamma_{p+1}} = \bigvee_{\gamma_1,\dots,\gamma_p} \tilde{z}_{\gamma_{p+q},\dots,\gamma_{p+1},\gamma_p,\dots,\gamma_1}. \tag{2}$$

Formulas (1), (2) are readily extended to unions of three or more sets and the corresponding simple vectors.

2. Let a primitive circuit P have s feedbacks $\varphi^1, \dots, \varphi^s$; n input buses $x^1, \dots, x^n$ and p output buses $f^1, \dots, f^p$, and let a nonprimitive circuit Q have r feedbacks $\psi^1, \dots, \psi^r$; m input buses $y^1, \dots, y^m$ and q output buses $g^1, \dots, g^q$. Suppose, further, that to the circuit P there corresponds the state matrix

$$A = \left\| a_{\alpha_s,\dots,\alpha_1;\alpha'_s,\dots,\alpha'_1}(x^1, \dots, x^n) \right\|$$

Fig. 1

Figure 1: Fig. 1

and the reaction matrix

$$L = \left\| l_{\alpha n,\dots,\alpha_1;\beta_p,\dots,\beta_1}(\varphi^1,\dots,\varphi^s) \right\|,$$

and to the circuit Q there correspond the state matrix

$$B = \left\| b_{\alpha_r,\dots,\alpha_1;\alpha'_r,\dots,\alpha'_1}(y^1,\dots,y^r) \right\|$$

and the reaction matrix

$$M = \left\| m_{\alpha m,\dots,\alpha_i;\beta_q,\dots,\beta_1}(\psi^1,\dots,\psi^r) \right\|.$$

If $\widetilde{\Phi}_{t+1}$ and $\widetilde{\Psi}_{t+1}$ are the simple state vectors, $\widetilde{X}_{t+1}$ and $\widetilde{Y}_{t+1}$ are the simple input vectors, and $\widetilde{F}_{t+1}$ and $\widetilde{G}_{t+1}$ are the simple output vectors of the circuits P and Q, respectively, then the relations (([1]), formulas (7) and (12)) hold:

$$\widetilde{\Phi}_{t+1} = \widetilde{\Phi}_t A(X_{t+1}); \qquad \widetilde{\Psi}_{t+1} = \widetilde{\Psi}_t B(Y_{t+1}); \tag{3}$$

$$\widetilde{F}_{t+1} = \widetilde{X}_{t+1} L(\Phi_t); \qquad \widetilde{G}_{t+1} = \widetilde{Y}_{t+1} M(\Psi_t). \tag{4}$$

where $X_{t+1} = (x^1_{t+1},\dots,x^n_{t+1})$; $Y_{t+1} = (y^1_{t+1},\dots,y^m_{t+1})$; $\Phi_t = (\varphi^1_t,\dots,\varphi^s_t)$, $\Psi_t = (\psi^1_t,\dots,\psi^r_t)$. We construct the composition R of the networks P and Q, putting

$$f^1_{t+1} = y^1_{t+1},\dots,f^v_{t+1} = y^v_{t+1}; \quad x^1_{t+1} = \vartheta^1_t,\dots,x^u_{t+1} = \vartheta^u_t;$$

$$\vartheta^1_{t+1} = g^1_{t+1},\dots,\vartheta^u_{t+1} = g^u_{t+1}; \quad v \leqslant \min(p,m); \quad u \leqslant \min(n,q). \tag{5}$$

The presence of delay lines $\vartheta^1,\dots,\vartheta^u$ is necessary so that contradictory expressions of the type $x = \bar{x}$ do not arise. If the functions $f^1,\dots,f^v$ do not depend on the variables $x^1,\dots,x^u$, or if the functions $g^1,\dots,g^u$ do not depend on the variables $y^1,\dots,y^v$, then the use of additional delay lines is not necessary. The composition R of the networks P and Q is shown in Fig. 1, where the connections corresponding to (5) are indicated by dotted lines*.

Fig. 1

The network R obtained in this way will have $k = m + n - u - v$ input buses, $l = p + q - u - v$ output buses, and $w = r + s + u$ feedback links. Introduce the notation:

$$z_{t+1}^1 = x_{t+1}^{u+1}, \dots, z_{t+1}^{n-u} = x_{t+1}^n; \quad z_{t+1}^{n-u+1} = y_{t+1}^{v+1}, \dots, z_{t+1}^k = y_{t+1}^m, \tag{6}$$

$$\sigma_t^1 = \varphi_t^1, \dots, \sigma_t^s = \varphi_t^s, \quad \sigma_t^{s+1} = \psi_t^1, \dots, \sigma_t^{s+r} = \psi_t^r, \quad \sigma_t^{s+r+1} = \vartheta_t^1, \dots, \sigma_t^w = \vartheta_t^u; \tag{7}$$

$$h_{t+1}^1 = f_{t+1}^{v+1}, \dots, h_{t+1}^{p-v} = f_{t+1}^p, \quad h_{t+1}^{p-v+1} = g_{t+1}^{u+1}, \dots, h_{t+1}^l = g_{t+1}^q. \tag{8}$$

The state of the network R at the moment $t+2$ is described by the simple vector $\widetilde{\Sigma}_{t+1} = (\tilde{\sigma}_{t+1}^0, \dots, \tilde{\sigma}_{t+1}^{2w-1})$. From relations (7) it follows that

$$\widetilde{\Sigma}_{t+1} = \Phi_{t+1} \widetilde{\Psi}_{t+1} \tilde{\theta}_{t+1}, \tag{9}$$

where $\tilde{\theta}_{t+1} = (\tilde{\vartheta}_{t+1}^0, \dots, \tilde{\vartheta}_{t+1}^{2u-1})$ is the simple vector describing, at the moment $t+2$, the state of the feedback links $\vartheta^1, \dots, \vartheta^u$.

By formulas (9), (3), (4), (5), using relations (1) and (2) and the notation (6), (7), (8), the elements of the state matrix $C = \|c_{\alpha_w,\dots,\alpha_1;\alpha'_w,\dots,\alpha'_1}(z_{t+1}^1, \dots, z_{t+1}^k)\|$ and of the reaction matrix $K = \|k_{\alpha_k,\dots,\alpha_1;\beta_l,\dots,\beta_1}(\sigma_t^1, \dots, \sigma_t^w)\|$ of the nonprimitive network R are computed:

$$c_{\alpha_w,\dots,\alpha_1;\alpha'_w,\dots,\alpha'_1}(z_{t+1}^1, \dots, z_{t+1}^k) =$$

$$= \bigvee_{\substack{\beta_p,\dots,\beta_1 \\ \gamma_{q-u},\dots,\gamma_1}} \alpha_{\alpha_s,\dots,\alpha_1;\alpha'_s,\dots,\alpha'_1}(\alpha_{s+r+1}, \dots, \alpha_w, z_{t+1}^1, \dots, z_{t+1}^{n-u}) \times$$

$$\times b_{\alpha_{s+r},\dots,\alpha_{s+1};\alpha'_{s+r},\dots,\alpha'_{s+1}}(\beta_1, \dots, \beta_v, z_{t+1}^{n-u+1}, \dots, z_{t+1}^k) \times$$

$$\times l_{z_{t+1}^{n-u},\dots,z_{t+1}^1,\alpha_w,\dots,\alpha_{s+r+1};\beta_p,\dots,\beta_1}(\alpha_1, \dots, \alpha_s) \times$$

$$\times m_{z_{t+1}^k,\dots,z_{t+1}^{n-u+1},\beta_w,\dots,\beta_1;\gamma_{q-u},\dots,\gamma_1,\alpha'_w,\dots,\alpha'_{s+r+1}}(\alpha_{s+1}, \dots, \alpha_{s+r}); \tag{10}$$

* Note that as delay lines $\vartheta^1, \dots, \vartheta^u$ one may use part of the lines ψ^i $(i = 1, \dots, r)$, and as f^j $(j = 1, \dots, v)$—par

$$k^{l}_{\alpha_k,\dots,\alpha_1;\,\beta_l,\dots,\beta_1}(\sigma^1_t,\dots,\sigma^w_t) =$$
$$= \bigvee_{\substack{\delta_v,\dots,\delta_1\\ \gamma_u,\dots,\gamma_1}} l_{\alpha_{n-u},\dots,\alpha_1;\,\sigma^w_t,\dots,\sigma^{s+r+1}_t;\,\beta_{p-v},\dots,\beta_1;\,\delta_v,\dots,\delta_1}(\sigma^1_t,\dots,\sigma^s_t)\times$$
$$\times m_{\alpha_k,\dots,\alpha_{n-u+1};\,\delta_v,\dots,\delta_1;\,\beta_l,\dots,\beta_{p-v+1};\,\gamma_u,\dots,\gamma_1}(\sigma^{s+1}_t,\dots,\sigma^{s+r}_t). \quad (11)$$

We shall call a composition of nonprimitive circuits **direct** if $u = 0$. We shall call a direct composition **successive** if, in addition, $m = p = v$. In the cases of direct and direct successive compositions, formulas (10) and (11) are simplified. Thus, for the case of a direct successive composition R of circuits P and Q, the number of input buses is $k = n$, the number of output buses is $l = q$, the number of feedbacks is $w = s + r$, and the corresponding formulas have the form

$$c_{\alpha_w,\dots,\alpha_1;\,\alpha'_w,\dots,\alpha'_1}(z^1_{t+1},\dots,z^n_{t+1}) = \bigvee_{\beta_p,\dots,\beta_1} a_{\alpha_s,\dots,\alpha_1;\,\alpha'_s,\dots,\alpha'_1}(z^1_{t+1},\dots,z^n_{t+1})\times$$
$$\times b_{\alpha_w,\dots,\alpha_{s+1};\,\alpha'_w,\dots,\alpha'_{s+1}}(\beta_1,\dots,\beta_p)\, l_{z^n_{t+1},\dots,z^1_{t+1};\,\beta_p,\dots,\beta_1}(\alpha_1,\dots,\alpha_s); \quad (12)$$

$$k_{\alpha_n,\dots,\alpha_1;\,\beta_q,\dots,\beta_1}(\sigma^1_t,\dots,\sigma^w_t) =$$
$$= \bigvee_{\gamma_p,\dots,\gamma_1} l_{\alpha_n,\dots,\alpha_1;\,\gamma_p,\dots,\gamma_1}(\sigma^1_t,\dots,\sigma^s_t)\, m_{\gamma_p,\dots,\gamma_1;\,\beta_q,\dots,\beta_1}(\sigma^{s+1}_t,\dots,\sigma^w_t). \quad (13)$$

Formula (13) corresponds to the product of the matrices L and M (1).

As an example, let us consider the direct successive composition of s trigger cells, i.e., of identical nonprimitive circuits, each of which has one input bus x^i, one output bus f^i, one feedback φ^i, and is described by the state matrix $A = \|a_{\alpha;\alpha'}(x^i_{t+1})\|$ and the reaction matrix $L = \|l_{\beta,\beta'}(\varphi^i_t)\|$:

$$A = \begin{pmatrix} \overline{x^i_{t+1}} & \underline{x^i_{t+1}} \\ x^i_{t+1} & x^i_{t+1} \end{pmatrix}; \qquad L = \begin{pmatrix} \underline{1} & 0 \\ \varphi^i_t & \varphi^i_t \end{pmatrix}; \qquad i = 1, 2, \dots, s. \quad (14)$$

The formulas for the elements of the state matrix and the reaction matrix of the composition easily follow from (12) and (13) and have the form

$$c_{\alpha_s,\dots,\alpha_1;\,\alpha'_s,\dots,\alpha'_1}(x^1_{t+1}) = \bigvee_{\beta_{s-1},\dots,\beta_1} a_{\alpha_1;\alpha'_1}(x^1_{t+1})\, a_{\alpha_2;\alpha'_2}(\beta_1)\cdots a_{\alpha_s;\alpha'_s}(\beta_{s-1})\times$$
$$\times l_{x'_{t+1};\,\beta_1}(\alpha_1)\, l_{\beta_1;\beta_2}(\alpha_2),\dots,l_{\beta_{s-2};\beta_{s-1}}(\alpha_{s-1}); \quad (15)$$

$$k_{\alpha;\alpha'}(\varphi_t^1, \dots, \varphi_t^s) =$$
$$= \bigvee_{\beta_{s-1},\dots,\beta_1} l_{\alpha;\beta_1}(\varphi_t^1)\, l_{\beta_1;\beta_2}(\varphi_t^2) \cdots l_{\beta_{s-2};\beta_{s-1}}(\varphi_t^{s-1})\, l_{\beta_{s-2};\alpha'}(\varphi_t^s). \tag{16}$$

Writing out, by these formulas, the state and reaction matrices of the composition, we obtain:

$$C = \begin{pmatrix} \overline{x_{t+1}^1} x_{t+1}^1 & 0 & \dots & 0 & 0 & \\ 0 & \overline{x_{t+1}^1} x_{t+1}^1 & \dots & 0 & 0 & \\ \cdot & \cdot & \cdot & \dots & \cdot & \cdot \\ 0 & 0 & 0 & \dots & \overline{x_{t+1}^1} x_{t+1}^1 & \\ x_{t+1}^1 & 0 & 0 & \dots & 0 & \overline{x_{t+1}^1} \end{pmatrix}; \qquad K = \begin{pmatrix} 1 & 0 \\ \overline{\varphi_t^1 \cdots \varphi_t^s} & \varphi_t^1 \cdots \varphi_t^s \end{pmatrix}. \tag{17}$$

The corresponding circuit (a binary-counting circuit) is often used-

is used in experimental physics (see, for example, ([2]), where numerous concrete schemes for its implementation on thyratrons, electronic tubes, etc. are given).

3. Let R be a nonprimitive circuit having w feedbacks $\sigma^1, \dots, \sigma^w$; n input buses $x^1, \dots, x^n$ and l output buses $f^1, \dots, f^l$;

$$C = \left\| c_{\alpha_w,\dots,\alpha_1;\alpha'_w,\dots,\alpha'_1}(x_{t+1}^1, \dots, x_{t+1}^n) \right\|, \quad K = \left\| k_{\alpha_n,\dots,\alpha_1;\beta_l,\dots,\beta_1}(\sigma_t^1, \dots, \sigma_t^w) \right\|$$

are the matrices of states and reactions of the circuit R. We indicate circuits P and Q whose composition is the circuit R. We give one of the possible decompositions of the circuit R into the subcircuits P and Q.

The circuit P has s feedbacks $\sigma^1, \dots, \sigma^s$; $n{+}w{-}s$ input buses $\sigma^{s+1}, \dots, \sigma^w, x^1, \dots, x^n$, and $p + s$ output buses $\sigma^1, \dots, \sigma^s, f^1, \dots, f^p$. The circuit Q has $w - s$ feedbacks $\sigma^{s+1}, \dots, \sigma^w$; $n + s$ input buses $\sigma^1, \dots, \sigma^s, x^1, \dots, x^n$, and $l - p + w - s$ output buses $\sigma^{s+1}, \dots, \sigma^w, f^{p+1}, \dots, f^l$. To the circuit P there correspond the state matrix

$$A = \left\| a_{\alpha_s,\dots,\alpha_1;\alpha'_s,\dots,\alpha'_1}(\sigma_t^{s+1}, \dots, \sigma_t^w, x_{t+1}^1, \dots, x_{t+1}^n) \right\|$$

and the reaction matrix

$$L = \left\| l_{\alpha_{n+w-s},\dots,\alpha_1;\beta_{p+s},\dots,\beta_1}(\sigma_t^1, \dots, \sigma_t^s) \right\|,$$

and to the circuit Q there correspond the state matrix

$$B = \|b_{\alpha_{w-s},\dots,\alpha_1;\alpha'_{w-s},\dots,\alpha'_1}(\sigma_t^1,\dots,\sigma_t^s,x_{t+1}^1,\dots,x_{t+1}^n)\|$$

and the reaction matrix

$$M = \|m_{\alpha_{n+s},\dots,\alpha_1;\beta_{l-p+w-s},\dots,\beta_1}(\sigma_t^{s+1},\dots,\sigma_t^w)\|,$$

computed by the formulas:

$$\alpha_{\alpha_s,\dots,\alpha_1;\alpha'_s,\dots,\alpha'_1}(\sigma_t^{s+1},\dots,\sigma_t^w,x_{t+1}^1,\dots,x_{t+1}^n) =$$

$$= \bigvee_{\substack{\gamma_{w-s},\dots,\gamma_1\\ \gamma'_{w-s},\dots,\gamma'_1}} c_{\gamma_{w-s},\dots,\gamma_1,\alpha_s,\dots,\alpha_1;\gamma'_{w-s},\dots,\gamma'_1,\alpha'_s,\dots,\alpha'_1}(x_{t+1}^1,\dots,x_{t+1}^n)\times$$

$$\times[\sigma_t^{s+1}]^{\gamma_1}\dots[\sigma_t^w]^{\gamma_{w-s}}; \tag{18}$$

$$b_{\alpha_{w-s},\dots,\alpha_1;\alpha'_{w-s},\dots,\alpha'_1}(\sigma_t^1,\dots,\sigma_t^s,x_{t+1}^1,\dots,x_{t+1}^n) = \tag{19}$$

$$= \bigvee_{\substack{\gamma_s,\dots,\gamma_1\\ \gamma'_s,\dots,\gamma'_1}} c_{\alpha_{w-s},\dots,\alpha_1;\gamma_s,\dots,\gamma_1;\alpha'_{w-s},\dots,\alpha'_1,\gamma'_s,\dots,\gamma'_1}(x_{t+1}^1,\dots,x_{t+1}^n)[\sigma_t^1]^{\gamma_1}\dots[\sigma_t^s]^{\gamma_s};$$

$$l_{\alpha_{n+w-s},\dots,\alpha_1;\beta_{p+s},\dots,\beta_1}(\sigma_t^1,\dots,\sigma_t^s) =$$

$$= \bigvee_{\gamma_{l-p},\dots,\gamma_1} k_{\alpha_{n+w-s},\dots,\alpha_{w-s+1};\gamma_{l-p},\dots,\gamma_1,\beta_s+p,\dots,\beta_{s+1}}(\sigma_t^1,\dots,\sigma_t^s,\alpha_1,\dots,\alpha_{w-s})\times$$

$$\times[\sigma_t^1]^{\beta_1}\dots(\sigma_t^s)^{\beta_s}; \tag{20}$$

$$m_{\alpha_{n+s},\dots,\alpha_1;\beta_{l-p+w-s},\dots,\beta_1}(\sigma_t^{s+1},\dots,\sigma_t^w) =$$

$$= \bigvee_{\gamma_p,\dots,\gamma_1} k_{\alpha_{n+s},\dots,\alpha_{s+1};\beta_{l-p+w-s},\dots,\beta_{w-s+1},\gamma_p,\dots,\gamma_1}(\alpha_1,\dots,\alpha_s,\sigma_t^{w-s+1},\dots,\sigma_t^w)\times$$

$$\times[\sigma_t^{s+1}]^{\beta_1}\dots(\sigma_t^w)^{\beta_{w-s}}. \tag{21}$$

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Note: Figure translations are in progress. See original paper for figures.