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Abstract

Full Text

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THEORY OF ELASTICITY

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SELF-SIMILAR PROBLEMS OF DYNAMIC BENDING OF PLATES

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The fundamental equation of bending of plates has the form ([1])

$$\frac{\partial^2 M_1}{\partial x^2} + 2\frac{\partial^2 M_{12}}{\partial x\,\partial y} + \frac{\partial^2 M_2}{\partial y^2} - \mu\frac{\partial^2 w}{\partial t^2} = 0. \tag{1}$$

Here $w(t,x,y)$ is the displacement perpendicular to the initial position of the plate; μ is the mass of a cylinder with base $dx\,dy$ on the middle surface; M_1, M_{12}, M_2 are bending moments, which in the general case are certain functions of the curvatures $\kappa_1 = \partial^2 w/\partial x^2$, $\kappa_{12} = \partial^2 w/\partial x\,\partial y$, $\kappa_2 = \partial^2 w/\partial y^2$, determined by the properties of the plate material.

Let us consider a class of self-similar solutions of equation (1) depending on two variables: $\xi = x/\sqrt{t}$, $\eta = y/\sqrt{t}$, i.e., $w = t\Phi_0(\xi,\eta)$. Since in this case $\kappa_1 = \partial^2\Phi_0/\partial\xi^2$, $\kappa_{12} = \partial^2\Phi_0/\partial\xi\,\partial\eta$, $\kappa_2 = \partial^2\Phi_0/\partial\eta^2$, the functions M_1, M_{12}, M_2 depend only on the variables ξ and η.

Consequently, equation (1) can be transformed to the form

$$\frac{\partial^2 M_1}{\partial \xi^2} + 2\frac{\partial^2 M_{12}}{\partial \xi\,\partial \eta} + \frac{\partial^2 M_2}{\partial \eta^2} + \frac{\mu}{4}\left[\xi\frac{\partial}{\partial\xi}\left(2\Phi_0 - \xi\frac{\partial\Phi_0}{\partial\xi} - \eta\frac{\partial\Phi_0}{\partial\eta}\right) + \eta\frac{\partial}{\partial\eta}\left(2\Phi_0 - \xi\frac{\partial\Phi_0}{\partial\xi} - \eta\frac{\partial\Phi_0}{\partial\eta}\right)\right] = 0. \tag{2}$$

A solution of the form under consideration is possessed, for example, by a number of problems on the impact upon a plate by a body moving with constant velocity.

In the particular case of impact on a plate, unbounded in both directions, by a body having one point ($x = y = 0$) of contact with the plate, a further simplification of the solution is possible.

In this case equation (1), in polar coordinates r, θ, assumes the form:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial M_1}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial r}(M_1-M_2)-\mu\frac{\partial^2 w}{\partial t^2}=0, \tag{3}$$

where the bending moments M_1 and M_2 depend only on the curvatures

$$\kappa_1=\frac{\partial^2 w}{\partial r^2}, \qquad \kappa_2=\frac{1}{r}\frac{\partial w}{\partial r}.$$

By virtue of the symmetry of this problem, the single variable on which the function Φ_0 depends will be $\zeta=r/\sqrt{t}$, whence equation (3) becomes the ordinary differential equation

$$\left[\frac{d}{d\zeta}\left(\zeta\frac{dM_1}{d\zeta}\right)+\frac{d}{d\zeta}(M_1-M_2)+\frac{\mu\zeta^2}{4}\left(\Phi_0''-\zeta(\Phi_0')\right)\right]=0 \tag{4}$$

of the third order with respect to the function Φ_0' (since $\kappa_1=\Phi_0''$, $\kappa_2=\Phi_0'/\zeta$).

For a linear dependence between stresses and strains, equation (4) is linear and can be integrated in elementary functions ([2]).

For the case of elastic-plastic deformations, equation (4), being nonlinear, has a different form in the regions of loading and unloading. We note that the boundary of these regions is determined by the equation $\zeta=\text{const}$; moreover, on it the condition of continuity of velocities, displacements, and bending moments must be satisfied, by analogy with the corresponding problem of impact on a beam ([3]).

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CITED LITERATURE

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[3] P. E. Duwer, D. S. Clark, N. F. Bohnenblust, *J. Appl. Mech.*, 17, No. 1 (1950) (transl. collected volume *Mechanics*, vol. 3, 1950).

Note: Figure translations are in progress. See original paper for figures.