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K. M. POLIVANOV

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Abstract

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ELECTRICAL ENGINEERING

K. M. POLIVANOV

DYNAMIC CHARACTERISTICS OF ELEMENTS OF ELECTRICAL CIRCUITS

(Presented by Academician M. A. Leontovich on 8 VII 1957)

1. The characteristics of nonlinear elements of electrical circuits in the quasistatic regime are represented by dependences of the type $y(x)$, for example $i(u)$, $\psi(i)$, $q(u)$, respectively for a semiconductor element, an inductance with a ferromagnetic core, and a capacitor with a ferroelectric. In the general case, the dependence between the same quantities under dynamic conditions is different. Therefore one sometimes speaks of a "dynamic characteristic." However, the form of the latter depends on the form of $x(t)$, i.e., on the nature of the time variation of the independent variable. This is obvious, since in real physical systems the transition from one state of equilibrium to another does not occur instantaneously, and the equation of the transient process

$$\varphi(x, y, \dot{x}, \dot{y}, \ddot{x}, ...) = 0 \tag{1}$$

contains, in addition to the two variables, their derivatives of various orders.

Equation (1) expresses a hypersurface in phase space with coordinates $x, y, \dot{x}, \dot{y}, ...$, which may be called the characteristic surface (C) or the surface of dynamic equilibrium. For a given external action $x(t)$, the point representing the state of the system traces on the surface C a trajectory whose projection onto the plane x, y is the "dynamic characteristic" * for the given pulse $x(t)$. Knowledge of the surface C makes it possible to find the response y of the element for any form of the pulse $x(t)$. Therefore only such a surface fully characterizes the dynamic properties of the element.

2. Just as, under quasistatic conditions, the curve $y(x)$ is determined experimentally as the characteristic of an element, under dynamic conditions the characteristic surface C can be determined experimentally. In the case of a linear system the surface C is a plane; in the case of a nonlinear system the surface is not planar. Of course, in the latter case a larger amount of experimental data is required; however, it is often sufficient to know only a small part of the whole surface, and in many cases for practical calculations one can restrict oneself to representing this surface in four- or even three-dimensional space, just as for slower processes one can restrict oneself to a static characteristic in the plane. In this case the surface, as it were, swells, since for different values of the higher derivatives the intersection of the hypersurface with three-dimensional space is represented by different surfaces.

By systematizing the results of observations, one can find points determining the position of the surface C in the region of interest to us, and carry out–

* In the case of a surface in the space $x, y, \dot{y}$, when its rise is steep near the intersection with the plane x, y, the deviation of the dynamic characteristic from the static one is small for not too large rates of variation of x and y; in this case the static characteristic is simply somewhat "smeared," "spread out."

determine to what extent the position of this surface is independent of the other coordinates of the phase space. A correct assumption about the form of the sought spatial characteristic can considerably facilitate its determination from experimental data.

In the presence of hysteresis, the entire space is permeated by surfaces of partial cycles, like a family of loops of partial cycles on a plane; therefore, in carrying out an experiment it is necessary to ensure motion along one chosen surface.

Even in the case where it is not possible to attain great accuracy, supplementing a plane characteristic at least to a three-dimensional surface nevertheless brings the characteristic of the element closer to the real one. It is also important to note the following circumstance: from the form of the characteristic surface, even without solving the corresponding equation, one can draw a number of conclusions about the influence of the form of the pulse $x(t)$ on the form of the response $y(t)$.

A refinement of the characteristic can be achieved by including additional imaginary circuit elements in order to approximate the characteristic of the real element, for example, in the manner set forth below (equation (7) and Fig. 3).

3. An example of a three-dimensional characteristic that should replace the quasistatic curve may be a non-planar surface described by a viscosity equation extended to a nonlinear system; for an inductance with a ferromagnetic core this equation has the form:

$$\frac{d\psi}{dt} = \alpha[\psi_{\mathrm{st}}(i) - \psi], \tag{2}$$

where $\psi_{\mathrm{st}}(i)$ is the quasistatic characteristic.

It is known from many experiments that the process of changing the polarization of ferroelectrics ([1]) and ferromagnetics ([2]) with a rectangular loop proceeds the faster, the more the external action exceeds the values corresponding to the critical field, i.e., that on average the rates $d\psi/dt$ or dq/dt are proportional to

$i - i_0$ or $u_c - u_0$, where i_0 and u_0 are the critical values at which the process of polarization change begins. What has been said can be taken into account by introducing into (2) an additional term:

$$\frac{d\psi}{dt} = \alpha[\psi_{\text{st}}(i) - \psi] + \beta(i - i_0). \tag{3}$$

For a rectangular characteristic $\psi_{\text{st}} = \pm\psi_0$ and $i_0 = \text{const}$; therefore, introducing new notation, instead of (3) we obtain:

$$u = \beta i - \alpha \int u\,dt + E, \tag{4}$$

where $u = d\psi/dt$ is the applied voltage, and E is a constant.

Restricting ourselves in the right-hand side of (4) to the first term alone, we obtain Sands' equation ([2]), derived by him from other considerations and applied to the calculation of the process of pulsed remagnetization of cores with a rectangular loop. Leaving only the middle term in the right-hand side, we arrive at a simple allowance for viscosity.

For a capacitor with a ferroelectric, instead of (4) one can write the analogous equation

$$i = \beta u - \alpha \int i\,dt + I. \tag{5}$$

Expressions (4) and (5) are the simplest linear approximations; for them it is easy to construct a linear electrical equivalent circuit. A better result can be obtained by composing nonlinear equations, of course at the cost of complicating the calculations.

4. For elements characterized by a rectangular loop with width $2x_0$ and height $2y_0$ (see Fig. 1), when considering the transition from $-y_0$ to $+y_0$

it is expedient to transfer the origin of coordinates to the point x_0, y_0 and to pass to the coordinates $\nu = x - x_0,\ \sigma = y_0 - y$. In this case, in the region $0 \geq \sigma \geq -2y_0,\ \nu \geq 0$, it is expedient to adopt for the derivative $d\sigma/dt = -dy/dt$ an expression that vanishes on the axes $\sigma = 0$ and $\nu = 0$. Among such expressions one may indicate, for example, the following:

$$-\frac{d\sigma}{dt} = \alpha\sigma^a\nu^b, \qquad -\frac{d\sigma}{dt} = \alpha\ \text{th}\, m\sigma\ \text{th}\, f\nu. \tag{6}$$

The equations given have the static characteristic as their limit and are easily integrated. However, they do not always reflect the features of the transient process observed in reality. A typical example of such a process is provided

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

by the oscillogram of current obtained by Merz ([1]). In Fig. 2 the curve M is the difference of the current curves given by Merz for switching on positive and negative voltages; this difference is equal to the current due to polarization (excluding the charge current of an ideal or "air"capacitance). On the oscillogram a gradual increase of the current is clearly visible. Such a character can be described only by a more complex equation containing, along with the first derivative, also the second derivative. The required complication of the equation is easily obtained from the equivalent circuit (Fig. 3)—the series connection of an inductance ensures the rise of current from zero. An analogous equivalent circuit can also be composed for a circuit with a ferromagnetic material; in it the series-connected $R,\ L,\ C$ are replaced by parallel ones.

Fig. 1

Fig. 2

Fig. 3

Putting $\nu = u_c - u_0$ and $\sigma = q_0 - q$, where u_c and q are the voltage and charge on the capacitor, from the circuit of Fig. 3 we obtain the equation:

$$u_r = -L\frac{d^2\sigma}{dt^2} - R\frac{d\sigma}{dt} + \nu + u_0. \tag{7}$$

Assuming that $d\sigma/dt$ is related to σ and ν by one of the equalities (6), we obtain a complete system of equations. Choosing the first of the equalities (6) and putting in it $a = b = 1$, after eliminating ν and integrating once with respect to t, we arrive at the equation

$$1 - s - m\ln s = \lambda\frac{ds}{d\theta} - \theta + \int_0^\theta w(\theta)\,d\theta, \tag{8}$$

where

Fig. 3

Figure 3: Fig. 3

$$s = \frac{\sigma}{2q_0}, \qquad \theta = \frac{u_0}{2q_0 R} t, \qquad w(\theta) = \frac{u_r(\theta)}{u_0},$$

m and λ are constants.

The curve of the reduced values of the current $i = -ds/dt$ as a function of the reduced-

time θ, computed from equation (8) for $m = 0.5$, $\lambda = 1$, and $w(t)$ equal to zero for $\theta < 0$ and equal to 2 for $\theta > 0$, is presented in Fig. 2; the calculated curve P is similar in character to the curve M, constructed from Merz' s experiments. Increasing the voltage in the computational equation leads to an acceleration of the process, as is observed in reality.

The last example shows a useful combination of nonlinearity, expressed by a three-dimensional characteristic surface, with the additional inclusion of linear equivalent elements. By passing to equivalent circuits, or by determining the role of each term in the equation, it is easy to take into account the influence of other circuit elements on the transient process—their presence usually leads only to a change in the coefficients of the differential equation.

5. The method of analyzing transient processes set forth here with examples from the field of electrical circuits can also be applied in other areas of physics, for example, to the study of impact mechanical strength; thus, of interest is the possibility of representing, by means of a characteristic surface, the dynamic strength of nylon threads investigated by Coleman ([3]); analogous characteristics were constructed by Feldbaum as applied to problems in control theory ([4]).

Moscow Power Engineering
Institute

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Note: Figure translations are in progress. See original paper for figures.