GEOPHYSICS

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Abstract

Full Text

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A METHOD FOR SHORT-RANGE FORECASTING OF METEOROLOGICAL ELEMENTS

The hypothesis of quasistaticity makes it possible to reduce the problem of short-range forecasting of the four principal meteorological elements—the three components of velocity u, v, w and the geopotential H—to the solution of a system of differential equations:

$$\frac{\partial u}{\partial t}+\frac{\partial H}{\partial x}-lv=-u\frac{\partial u}{\partial x}-v\frac{\partial u}{\partial y}-\tau\frac{\partial u}{\partial \zeta}; \tag{1}$$

$$\frac{\partial v}{\partial t}+\frac{\partial H}{\partial y}+lu=-u\frac{\partial v}{\partial x}-v\frac{\partial v}{\partial y}-\tau\frac{\partial v}{\partial \zeta}; \tag{2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial \tau}{\partial \zeta}=0; \tag{3}$$

$$\zeta^2\frac{\partial^2 H}{\partial \zeta\partial t}+c^2\tau=-\zeta\left(u\frac{\partial^2 H}{\partial x\partial \zeta}+v\frac{\partial^2 H}{\partial y\partial \zeta}\right). \tag{4}$$

Here the independent variables are the horizontal coordinates x and y, the reduced pressure $\zeta = p/P$ (p is pressure, P is pressure at sea level), and time t; l is the Coriolis parameter; $c^2 = \alpha RT_1$; R is the gas constant; T_1 is the mean temperature; $\alpha = (\gamma_a - \gamma)R/g$ (g is the acceleration of gravity, γ is the vertical temperature gradient, γ_a is the adiabatic gradient). In this case one may assume that the function τ is related to the vertical velocity w by the relation

$$\tau=\frac{\zeta}{RT_1}\left(\frac{\partial H}{\partial t}-gw\right). \tag{5}$$

As boundary conditions it is assumed that

$$\zeta w = 0 \quad \text{for } \zeta = 0 \tag{6}$$

(the atmosphere neither leaves nor enters at its upper boundary), and that

$$w = 0 \quad \text{for } \zeta = 1 \tag{7}$$

(the earth’ s surface is horizontal).

Let us introduce the potential and solenoidal parts for the horizontal velocities u, v:

$$u = -\frac{\partial \psi}{\partial y} + \frac{\partial \varphi}{\partial x}, \qquad v = \frac{\partial \psi}{\partial x} + \frac{\partial \varphi}{\partial y}. \tag{8}$$

Differentiating equation (2) with respect to x, and equation (1) with respect to y, and subtracting the results, we obtain

$$-\frac{\partial \Delta \psi}{\partial t} + l \Delta \varphi = -B_{\Omega}, \tag{9}$$

where B_{Ω} contains the advection of the vortex Ω ($\Omega = \partial v/\partial x - \partial u/\partial y = \Delta \psi$; $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$) and has the form:

$$B_{\Omega} = u\frac{\partial \Omega}{\partial x} + v\frac{\partial \Omega}{\partial y} + \tau\frac{\partial \Omega}{\partial \zeta} + \Delta\varphi \cdot \Omega + \beta v + \frac{\partial \tau}{\partial x}\frac{\partial v}{\partial \zeta} - \frac{\partial \tau}{\partial y}\frac{\partial u}{\partial \zeta}$$

($\beta = dl/dy$). In an analogous way, differentiating (1) with respect to x, (2) with respect to y, and adding the results, we obtain:

$$\frac{\partial \Delta \varphi}{\partial t} + \Delta H - l\Delta\psi = -B_D, \tag{10}$$

where B_D contains the advection of the divergence D ($D = \partial u/\partial x + \partial v/\partial y = \Delta\varphi$) and is written as follows:

$$B_D = u\frac{\partial D}{\partial x} + v\frac{\partial D}{\partial y} + \tau\frac{\partial D}{\partial \zeta} +$$

$$+ \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + 2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + \beta u + \frac{\partial \tau}{\partial x}\frac{\partial u}{\partial \zeta} + \frac{\partial \tau}{\partial y}\frac{\partial v}{\partial \zeta}.$$

Finally, eliminating τ from (3) and (4) (we regard the quantity c^2 as constant), we obtain:

$$\frac{\partial}{\partial \zeta}\zeta^2\frac{\partial^2 H}{\partial \zeta\,\partial t} - c^2\Delta\varphi = R - \frac{\partial \zeta B_T}{\partial \zeta}, \tag{11}$$

where B_T is the horizontal advection of temperature T $\left(T = -\frac{\zeta}{R}\frac{\partial H}{\partial \zeta}\right)$:

$$B_T = u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = -\frac{\zeta}{R}\left(u\frac{\partial^2 H}{\partial x\,\partial\zeta} + v\frac{\partial^2 H}{\partial y\,\partial\zeta}\right).$$

The right-hand sides of (9), (10), (11) can be expressed through φ, ψ, H. As τ one may, according to (3), (6), and (8), take

$$\tau = -\int \Delta\varphi\, d\zeta. \tag{12}$$

The vertical velocity w is related, according to (4) and (5), to φ, ψ, H by the relation

$$\zeta\frac{\partial^2 H}{\partial\zeta\,\partial t} + \alpha\frac{\partial H}{\partial t} = \alpha g w + R B_T. \tag{13}$$

Our system contains three differentiations with respect to time. We shall assume that at the initial moment three functions φ, H, ψ are known. Let

$$(\varphi)_{t=0} = \varphi_0(x, y, \zeta); \qquad (\psi)_{t=0} = \psi_0(x, y, \zeta); \qquad (H)_{t=0} = H_0(x, y, \zeta). \tag{14}$$

To solve the problem of determining φ, ψ, and H from the system (9), (10), (11), under the boundary conditions (6), (7) and the initial conditions (14), we shall temporarily regard B_Ω, B_D, B_T as known functions of x, y, ζ, t. Then our system will be linear. From it we determine the functions φ, ψ, and H. To this end we first eliminate the functions φ and ψ and obtain an equation for H:

$$\left(\frac{\partial^2}{\partial t^2} + l^2\right)\frac{\partial}{\partial\zeta}\zeta^2\frac{\partial^2 H}{\partial\zeta\,\partial t} + c^2\Delta\frac{\partial H}{\partial t} = R\left(\frac{\partial^2}{\partial t^2} + l^2\right)\frac{\partial\zeta B_T}{\partial\zeta} - c^2\left(lB_\Omega + \frac{\partial B_D}{\partial t}\right). \tag{15}$$

It will be more convenient, however, to seek not H, but w: for w we have simple boundary conditions (6) and (7). Guided by the relation (13) existing between w and H (B_T is still regarded as a prescribed function), we easily derive from (15) an equation for w. It will have the form

$$\left(\frac{\partial^2}{\partial t^2} + l^2\right)\zeta^2\frac{\partial^2\zeta w}{\partial\zeta^2} + c^2\Delta\zeta w = -\zeta F, \tag{16}$$

where

$$F = \frac{RT_1}{g}\left[l\left(\alpha + \zeta\frac{\partial}{\partial\zeta}\right)B_\Omega + R\Delta B_T - \frac{1}{T_1}\left(\frac{\partial^2}{\partial t^2} + l^2\right)\frac{\partial\zeta B_T}{\partial\zeta} + \left(\alpha + \zeta\frac{\partial}{\partial\zeta}\right)\frac{\partial B_D}{\partial t}\right].$$

The solution of an equation of type (16) under the boundary conditions (6) and (7) is known (see $^{(1)}$, where the solution of the homogeneous equation under the boundary conditions (6) and (7) is given, and $^{(2)}$, where the solution of the equation with a right-hand side is given):

$$\zeta w = \frac{2}{2\pi l}\left[\frac{\partial}{\partial t}\int_0^{2\pi}\int_0^1\int_0^{2lt} g_1\zeta' w_0\,dr\,d\zeta'\,d\delta + \int_0^{2\pi}\int_0^1\int_0^{2lt} g_1\zeta' w_1\,dr\,d\zeta'\,d\delta + \int_0^t\int_0^{2\pi}\int_0^1\int_0^{2l(t-t')} \tilde{g}F\,dr\,d\zeta'\,d \right. \tag{17}$$

Here

$$w_0 = (w)_{t=0}, \qquad w_1 = \left(\frac{\partial w}{\partial t}\right)_{t=0}, \qquad r^2 = \frac{l^2}{c^2}\left[(x-x')^2 + (y-y')^2\right],$$

$$x' = x + \frac{c}{l}r\cos\delta, \qquad y' = y + \frac{c}{l}r\sin\delta,$$

$$g_1 = \frac{1}{2\zeta'}\sqrt{\frac{\zeta}{\zeta'}}\left(\frac{1}{4} - \frac{\partial^2}{\partial a^2}\right)\Bigg|_{a=\ln\frac{\zeta}{\zeta'}}^{a=\ln\frac{1}{\zeta\zeta'}} J_0\left(\sqrt{\frac{l^2t^2}{r^2} - \frac{1}{4}}\sqrt{r^2+a^2}\right),$$

$$\tilde{g} = \frac{1}{2}\sqrt{\frac{\zeta}{\zeta'}}\Bigg|_{a=\ln\frac{1}{\zeta\zeta'}}^{a=\ln\frac{\zeta}{\zeta'}} J_0\left(\sqrt{\frac{l^2(t-t')^2}{r^2} - \frac{1}{4}}\sqrt{r^2+a^2}\right).$$

In this case w_0 and w_1 are expressed without difficulty in terms of φ_0, ψ_0, H_0, and also in terms of B_T and B_D.

Carrying out integration by parts and several other transformations, we can reduce the expression for w to the form

$$\begin{aligned}\alpha g\zeta w = -RB_T + \frac{1}{2\pi l^2}\frac{\partial}{\partial t}\Bigg\{ & c^2\frac{\partial}{\partial t}\int_0^{2\pi}\int_0^1\int_0^{2lt} G_0\Delta\varphi_0\,r\,dr\,d\zeta'\,d\delta + c^2\int_0^{2\pi}\int_0^1\int_0^{2lt} G_0\Delta(\psi_0 - H_0)\,r\,dr\,d\zeta'\,d\delta \\ & - \int_0^t\int_0^{2\pi}\int_0^1\int_0^{2l(t-t')}\left[lc^2GB_\Omega + R\zeta'\frac{\partial}{\partial\zeta'}\left(l^2G + \frac{\partial^2 G}{\partial t^2}\right)B_T - c^2\frac{\partial G}{\partial t}B_D\right]r\,dr\,d\zeta'\,d\delta\,dt'\Bigg\},\end{aligned} \tag{18}$$

where

$$G = \frac{1}{2\sqrt{\zeta\zeta'}} \left(\frac{1}{2} - \alpha + \zeta' \frac{\partial}{\partial \zeta'} \right) \Bigg|_{a=\ln \frac{1}{\zeta\zeta'}}^{a=\ln \frac{\zeta}{\zeta'}} S(r, a, t - t'); \qquad G_0 = (G)_{t'=0},$$

and

$$S(r, a, t - t') = \int_0^{\sqrt{\frac{l^2(t-t')^2}{r^2} - \frac{1}{4}}} \frac{J_0\left(\lambda\sqrt{r^2 + a^2}\right) \lambda \, d\lambda}{\sqrt{\lambda^2 + 1/4}}. \tag{19}$$

As soon as w has been determined, equation (13) makes it possible to find $\partial H/\partial t$, and then H; the solution for H bounded at $\zeta = 0$ has the form

$$\begin{aligned} H = H_0 + \frac{1}{2\pi l^2} \Bigg\{ &- \frac{\partial}{\partial t} \int_0^{2\pi} \int_0^1 \int_0^{2lt} G_0^H c^2 \Delta\varphi_0 \, r \, dr \, d\zeta' \, d\delta \\ &- \int_0^{2\pi} \int_0^1 \int_0^{2lt} G_0^H c^2 \Delta(\psi_0 - H_0) \, r \, dr \, d\zeta' \, d\delta \\ &+ \int_0^t \int_0^{2\pi} \int_0^1 \int_0^{2l(t-t')} \left[c^2 l G^H B_\Omega + \zeta' \frac{\partial}{\partial \zeta'} \left(\frac{\partial^2 G^H}{\partial t^2} + l^2 G^H \right) R B_T - c^2 \frac{\partial G^H}{\partial t} B_D \right] r \, dr \, d\zeta' \, d\delta \end{aligned} \tag{20}$$

where

$$G^H = \frac{1}{2\sqrt{\zeta\zeta'}} \left[(S)_{a=\ln \frac{\zeta}{\zeta'}} + (S)_{a=\ln \frac{1}{\zeta\zeta'}} + (1 - 2\alpha) e^{-\left(\frac{1}{2} - \alpha\right) \ln \frac{1}{\zeta\zeta'}} \int_{\ln \frac{1}{\zeta\zeta'}}^{\infty} e^{\left(\frac{1}{2} - \alpha\right) a} S \, da \right], \qquad G_0^H = (G^H)_{t'=0}.$$

After H has been found, by simple differentiation we find $c^2\Delta\varphi$ from (11), and then, from (9), $\Delta\psi$ by quadratures with respect to time.

Formula (20) and analogous formulas for φ and ψ can be used for forecasting meteorological elements. The integrals containing $\Delta\varphi_0$ and $\Delta(\psi_0 - H_0)$ are of no interest for forecasting: they either decay rapidly with time or become stationary small quantities and may be discarded. In making a forecast, the entire time interval of interest must be divided into small subintervals, within each of which B_Ω, B_T, B_D are to be regarded as approximately constant in time, these latter quantities being determined at the end of each subinterval with the aid of (20) and of analogous expressions for φ and ψ.

Formula (20) is a generalization of the widely known prognostic formula of N. I. Buleev and G. I. Marchuk, obtained under the assumption of quasigeostrophy. In particular, as $t \to \infty$ our function G^H passes into the Green' s function that appears in the advection of vorticity in the aforementioned prognostic formula.

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Note: Figure translations are in progress. See original paper for figures.

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