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Abstract

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On the Stability of Solutions of Some Differential Boundary-Value Problems with Partial Derivatives in a Banach Space

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M. G. Krein ([1]) was the first to consider the question of boundedness of solutions of differential equations in a Banach space. M. A. Rutman, in ([2]), proposed a method that makes it possible to study the question of stability of solutions of certain operator equations, and applied it in ([3–5]) to the qualitative investigation of solutions of certain linear differential equations with partial derivatives. All of them belong to equations of the form

$$\frac{\partial^{p_1+p_2+\cdots+p_n}u}{\partial t_1^{p_1}\partial t_2^{p_2}\cdots\partial t_n^{p_n}} - \sum A_{q_1 q_2 \cdots q_n}\frac{\partial^{q_1+q_2+\cdots+q_n}u}{\partial t_1^{q_1}\partial t_2^{q_2}\cdots\partial t_n^{q_n}} = v,$$

where $p_i \geqslant g_i$, $\sum_{i=1}^{n} p_i > \sum_{i=1}^{n} q_i$. By applying to both sides of such an equation a product of Volterra integration operators, it is reduced to an equation with continuous operators, considered in ([2]).

We have considered several boundary-value problems that do not belong to the indicated type.

Let us consider in the half-plane $Q:\ -\infty < x < \infty,\ t \geqslant 0$ the differential boundary-value problems:

$$\frac{\partial u}{\partial t} - a\frac{\partial u}{\partial x} - A(x,t)u = f(x,t), \qquad u(x,0) = \varphi(x); \tag{1}$$

$$\frac{\partial u}{\partial t} - a^2\frac{\partial^2 u}{\partial x^2} - A(x,t)u = f(x,t), \qquad u(x,0) = \varphi(x) \quad (a \neq 0); \tag{2}$$

$$\frac{\partial^2 u}{\partial t^2} + 2\alpha\frac{\partial u}{\partial t} - a^2\frac{\partial^2 u}{\partial x^2} - A(x,t)u = f(x,t), \qquad u(x,0) = \varphi(x), \qquad u'_t(x,0) = \psi(x), \tag{3}$$

where $u(x,t)$ is the unknown function; $f(x,t)$, $\varphi(x)$, $\psi(x)$ are given functions with values in a complex Banach space $\widetilde{E}$, defined and continuous in the half-plane Q; α and a are real numbers ($\alpha > 0$); $A(x,t)$ is a continuous operator-valued function with values in the normed ring R of all linear continuous operators acting in $\widetilde{E}$.

Regarding the operator-function $A(x,t)$ we shall assume the following:

1°. The family $\{A(x,t)\}$ is compact.

2°. The operator-function $A(x,t)$ has "small variation at t-infinity" : for a sufficiently small number $\varepsilon > 0$ there exists a $T > 0$ such that for any x_1 and x_2, from the conditions $t_1 > T,\ t_2 > T,\ |t_1 - t_2| \leqslant 1$ it follows that

$$\|A(x_1,t_1) - A(x_2,t_2)\| < \varepsilon.$$

We shall call an operator $A_\omega \in R$ an ω_t-limit operator for the operator-function $A(x,t)$ if there exists a sequence of points $(x_n,t_n) \in Q,\ t_n \to \infty$, such that

$$\lim_{n\to\infty} A(x_n,t_n) = A_\omega.$$

We shall call the boundary-value problems (1), (2) stable if, to any functions $f(x,t)$ and $\varphi(x)$ uniformly bounded in Q, there corresponds a solution uniformly bounded in Q.

We shall call the boundary-value problem (3) stable if, to any functions $f(x,t)$, $\varphi(x)$, $\varphi'(x)$, and $\psi(x)$ uniformly bounded in Q, there corresponds a solution uniformly bounded in Q.

Theorem 1. *In order that the boundary-value problems (1) and (2) be stable, it is necessary and sufficient that the spectra of all ω_t-limit operators for the operator-function $A(x,t)$ lie inside the left half-plane.*

Theorem 2. *In order that the boundary-value problem (3) be stable, it is necessary and sufficient that the spectra of all ω_t-limit operators for the operator-function $A(x,t)$ lie inside the region bounded by the parabola $\eta^2 = -4\alpha^2\xi$.*

The boundary-value problem (3) leads to the equation of electrical oscillations in an infinite conductor.

Let us note that, from Theorem 1, for the boundary-value problem (1) with $a = 0$, one obtains the well-known result of M. G. Krein (see ([1]), Theorem 3), refined by M. A. Rutman in ([4]). In the finite-dimensional case this refinement was made by N. Ya. Lyashchenko ([6]).

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Note: Figure translations are in progress. See original paper for figures.