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Abstract

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HYDROMECHANICS

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ON MEDIA ADMITTING ONE-DIMENSIONAL MOTIONS WITH HOMOGENEOUS DEFORMATION

(Presented by Academician L. I. Sedov, 23 I 1958)

One-dimensional motions with homogeneous deformation, i.e., motions satisfying the condition

$$r = \mu(t) r_0 \tag{1}$$

(where r is the distance of a particle either to some plane, or to an axis, or to a point, and r_0 is the same distance at $t = 0$), were considered in works ([1–5]), the medium being regarded as a perfect gas. Below an answer is given to the question: what equation of state must a medium obey in order to admit one-dimensional motions with homogeneous deformation?

Let us consider two cases: the motion satisfies either the adiabaticity condition $\partial S/\partial t = 0$, or the condition $\partial T/\partial r = 0$ (T is the temperature). In the first case we shall seek the equations of state in the form $p = p_1(\rho, S)$, and in the second case in the form $p = p_2(\rho, T)$.

If external forces are absent, and the internal stresses reduce to pressure, as well as in certain other cases (as, for example, in the presence of gravitation ([2])), it follows from the momentum equation under condition (1) that

$$\frac{1}{\rho_0} \frac{\partial p}{\partial r_0} = k(t) r_0, \tag{2}$$

where ρ_0 is the initial density.

Equality (2) will be the starting point in the subsequent considerations. From equality (2), taking $p(r_0, t) = p(p_0, t)$ ($p_0(r_0)$ is the initial pressure), we obtain

$$\frac{\partial p}{\partial p_0} = \frac{k(t)}{k(0)} = f(t),$$

whence it follows that

$$p(r_0, t) = f(t) p_0(r_0) + \varphi(t). \tag{3}$$

Denoting $\rho/\rho_0 = \alpha(t)$ and introducing $\alpha(t)$ and $\rho_0(r_0)$ as new variables instead of t and r_0, we rewrite equality (3) as follows:

$$\begin{aligned} p_1(\rho, S) &= p_1(\alpha\rho_0,\ S(\rho_0)) = f_1(\alpha) p_{01}(\rho_0) + \varphi_1(\alpha), \\ p_2(\rho, T) &= p_2(\alpha\rho_0,\ T(\alpha)) = f_2(\alpha) p_{02}(\rho_0) + \varphi_2(\alpha). \end{aligned} \tag{4}$$

Introducing the inverse functions $\rho_0(S)$ and $\alpha(T)$, we finally obtain the equations of state in the following form:

$$\begin{aligned} p_1(\rho, S) &= f_1\left(\frac{\rho}{\rho_0(S)}\right) p_{01}(\rho_0(S)) + \varphi_1\left(\frac{\rho}{\rho_0(S)}\right), \\ p_2(\rho, T) &= f_2(T) p_{02}\left(\frac{\rho}{\alpha(T)}\right) + \varphi_2(T), \end{aligned} \tag{5}$$

where $\rho_0(S)$ and $\alpha(T)$ are prescribed, while f, p_0, and φ are arbitrary functions.

Now let us find, among these equations, those equations of state which admit one-dimensional motions with homogeneous deformation for arbitrary functions $\rho_0(S)$ and $\alpha(T)$. To this end, let us first consider the case $S(\rho_0) = \text{const.}$ and $T(\alpha) = \text{const.}$ Then p in equalities (4) does not depend on the second arguments. Applying to both sides of these equalities the operator $\dfrac{\partial^2}{\partial\rho_0 \partial\alpha} \ln \dfrac{1}{\alpha} \dfrac{\partial}{\partial\rho_0}$, we obtain:

$$\left(\frac{p''}{p'}\right)' \rho + \frac{p''}{p'} = 0,$$

where the prime denotes differentiation with respect to ρ. After integration these equalities take the form:

$$\begin{aligned} p_1(\rho, S) &= A_1(S) \rho^{\gamma_1} + B_1, \\ p_2(\rho, T) &= A_2(T) \rho^{\gamma_2} + B_2(T), \end{aligned} \tag{6}$$

where A_1, γ_1, and B_1, generally speaking, are functions of S, and A_2, γ_2, and B_2 are functions of T. But comparing these expressions with equalities (4), we note that $\gamma_1 = \text{const.}$, $\gamma_2 = \text{const.}$, $B_1 = \text{const.}$ If the deformation is homogeneous, then by direct verification it is easy to see that, under these conditions, equalities (6) indeed satisfy condition (3) for arbitrary functions $S(\rho_0)$ and $T(\alpha)$.

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References

[1] L. I. Sedov, DAN, **90**, No. 5 (1953).
[2] M. L. Lidov, DAN, **97**, No. 3 (1954).
[3] A. G. Kulikovskii, DAN, **114**, No. 5 (1957).
[4] I. M. Yavorskaya, DAN, **114**, No. 5 (1957).
[5] J. B. Keller, Quart. Appl. Math., **14**, No. 2 (1956).

Note: Figure translations are in progress. See original paper for figures.