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Abstract

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HYDROMECHANICS

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POTENTIAL STEADY RELATIVISTIC GAS FLOWS

(Presented by Academician L. I. Sedov, 14 VI 1958)

One-dimensional steady relativistic gas flows have recently been considered by K. P. Stanyukovich ([1]) *.

In the present note, the general case of a spatial steady potential relativistic flow is considered and, in particular, the especially simple case of an ultrarelativistic gas.

In the case of a barotropic flow without friction and thermal conductivity, flows are possible in which the so-called pseudovelocity possesses a potential ([2,3]). By pseudovelocity we mean the quantity

$$v^i = \frac{w}{\rho} u^i = J u^i, \tag{1}$$

where u^i is the relativistic 4-velocity; ρ is the rest mass-energy per unit proper volume; w is the relativistic heat function per unit proper volume. The quantity J, consequently, is the heat function per unit rest energy-mass, so that $J \geqslant 1$.

In the case of potential flow we have

$$v_i = \frac{\partial \varphi}{\partial x^i}. \tag{2}$$

The pseudovelocity potential satisfies the differential equation

$$\left[g^{ik} + (\bar{a}^{-2} - 1) u^i u^k\right] \frac{\partial^2 \varphi}{\partial x^i \partial x^k} = 0 \ \ **, \tag{3}$$

where $\bar{a}$ is the relativistic speed of sound (the speed of light is here taken as unity). Owing to the constancy of the specific entropy, the quantity $\bar{a}$ is a function of the single quantity J, which is determined by the equation

$$J^2 = g_{ik} v^i v^k. \tag{4}$$

In the case of a steady gas flow we have

$$\frac{\partial v_i}{\partial x^0} = 0 \quad \text{or} \quad \frac{\partial v_0}{\partial x^i} = 0 \quad (i = 0, 1, 2, 3).$$

* The principal results of this work were obtained independently by me and A. A. Aryanov and were reported at the scientific conference of Kabardino-Balkarian State University on 23 II 1958. The case of plane-parallel steady potential flow was considered as early as 1954–55 in work carried out under my supervision by S. V. Danov. Unfortunately, S. V. Danov did not publish these results.

** In cases of general relativity and curvilinear coordinates, instead of $\partial^2\varphi/\partial x^i \partial x^k$ one must use the corresponding absolute derivative.

In other words,

$$v_0 = \text{const}. \tag{5}$$

This is the relativistic Bernoulli equation ([4]). The ordinary velocity, i.e. the 3-vector $\hat{u}_i = dx^i/dx^0$, is then determined by the equation

$$\hat{u}_i = -\frac{v_i}{v_0} \quad (i = 1, 2, 3). \tag{6}$$

Thus, the ordinary velocity $\hat{u}_i$ in the flows under consideration has a potential, which in what follows we shall denote by $\hat{\varphi}$.

We shall henceforth restrict ourselves to a Lorentz coordinate system ($g_{00} = -g_{11} = -g_{22} = -g_{33} = 1,\ g_{ik} = 0$ for $i \neq k$). Then equation (3) takes the form

$$\sum_{i=1}^{3} \left[(1 - \hat{u}^2) - (\bar{a}^{-2} - 1)\hat{u}_i^2\right] \frac{\partial^2 \hat{\varphi}}{\partial x_i^2} - (\bar{a}^{-2} - 1) \sum_{i,k=1 (i \neq k)}^{3} \hat{u}_i \hat{u}_k \frac{\partial^2 \hat{\varphi}}{\partial x_i \partial x_k} = 0, \tag{7}$$

where

$$\hat{u}_i = \frac{\partial \hat{\varphi}}{\partial x^i}, \qquad \hat{u}^2 = \hat{u}_1^2 + \hat{u}_2^2 + \hat{u}_3^2. \tag{7a}$$

In the case of an ultrarelativistic gas, where $\bar{a}^2 = 1/3$, we have

$$\sum_{i=1}^{3} \left[\frac{1}{2}(1 - \hat{u}^2) - \hat{u}_i^2\right] \frac{\partial^2 \hat{\varphi}}{\partial x_i^2} - \sum_{i,k=1 (i \neq k)}^{3} \hat{u}_i \hat{u}_k \frac{\partial^2 \hat{\varphi}}{\partial x_i \partial x_k} = 0. \tag{8}$$

This equation coincides completely with the corresponding equation for a classical ideal gas if the ratio of specific heats is taken to be $\varkappa = 2$. Indeed, the equation mentioned has the form

$$\sum_{i=1}^{3}\left[\frac{\varkappa+1}{2}(1-\hat{u}^2)-\hat{u}_i^2\right]\frac{\partial^2\hat{\varphi}}{\partial x_i^2}-\sum_{i,k=1(i\neq k)}^{3}\hat{u}_i\hat{u}_k\frac{\partial^2\hat{\varphi}}{\partial x_i\partial x_k}=0,$$

* if the speed of outflow into vacuum is taken as unity.

The present result remains valid in the case of a photon gas, for which $\rho = 0$. In this case it can be obtained directly from the energy-momentum equations

$$\frac{\partial T^{ik}}{\partial x^k}=0,$$

where $T^{ik}=\frac{e}{3}(4u^iu^k-g^{ik})$ (e is the internal energy per unit proper volume).

In the case of plane-parallel flow it follows from this that Chaplygin' s theory of gas jets ([5]) can be applied to an ultrarelativistic and photon gas, taking $\varkappa = 2$.

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5. S. A. Chaplygin, *On Gas Jets*, Collected Works, **2**, 1948.

Note: Figure translations are in progress. See original paper for figures.