Reports of the Academy of Sciences of the USSR

1958

SovietRxiv

Abstract

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Reports of the Academy of Sciences of the USSR
1958, Volume 123, No. 5

PHYSICS

V. T. KHOZYAINOV

HEAT TRANSFER BETWEEN THE NORMAL AND SUPERFLUID PHASES OF LIQUID HELIUM

(Presented by Academician N. N. Bogolyubov on 18 VII 1958)

1. V. P. Peshkov (1) succeeded in observing a new phenomenon in the physics of liquid helium: a visible boundary between two modifications of liquid helium (superfluid and normal), under the condition that a steady heat flux passes through this boundary. Although under ordinary conditions the phase transition in helium between these two modifications is a second-order transition, in the present case a discontinuity (jump) of density and temperature was observed at the boundary, and its dependence on the density of the heat flux was measured. Thus, here we are dealing with the problem of transfer of thermal energy from normal to superfluid helium (or from a "classical" to a "quantum" liquid), and also with the existence of a stable boundary between these modifications, despite the obvious presence of turbulence (turbulent thermal conductivity) in normal helium.

2. The mechanisms of heat conduction on the two sides of the boundary are fundamentally different (superfluid helium is also a superconductor of heat). In the normal phase ordinary classical (convective-turbulent) thermal conductivity is realized ($q = -\varkappa_{\text{turb}} \nabla T$), while the heat flux in superfluid helium is a flux of elementary excitations, with the thermal resistance being practically zero and the temperature ($T_0 < T_\lambda$) remaining constant along the whole of helium II.

From the phenomenological point of view, at the boundary under consideration, in the stationary case, there occur continuous absorption of the superfluid part ρ_s and generation of the normal part ρ_n of the density of helium II ($\rho = \rho_n + \rho_s$, $\rho_n v_n = -\rho_s v_s$). Consequently, multiple generation of excitation quanta takes place ("rotons," which at the temperatures under consideration, close to T_λ, play the principal role). Formally, the heat flux through the boundary may be represented by the expression

$$q = a(\Delta T)^\alpha, \qquad (1)$$

where $\Delta T = T - T_0$ is the measured temperature jump, with $T > T_\lambda$ being the mean temperature of the normal phase near the interface; the exponent α may be regarded as an analogue of the order (generally speaking, fractional) of a heterogeneous "reaction." The theory must determine the parameters of formula (1).

The stationary state of superfluid helium in our case is determined by the maximum of the entropy of helium II $S(T, v)$ (as a function of temperature and "relative" velocity $v = v_n - v_s$) at a given heat flux $q = \text{const}$. Using the known expressions for ρ_n, the entropy, and the heat flux in superfluid helium $^{(2)}$, we find that $\delta T = T_\lambda - T_0 \simeq 4 \cdot 10^{-3} q^{2/3}$, where q is expressed in watts per 1 cm^2. Thus, the stationary temperature T_0 is lower the greater the heat flux passing through the phase boundary.

3. As a mechanism for the multiple generation of quasiparticles—rotons—one may imagine the formation in the boundary region of co-

composite systems possessing a higher energy density than the surrounding medium, and arising as a result of "mixing" of a hydrodynamically unstable boundary (drops of "hot" normal helium in "cold" superfluid helium). These systems subsequently decay, passing into one of the possible states characterized by the number of quasiparticles produced. The probability of such an event, as well as the mean number of rotons formed in one decay event of a given energy, were found in (^{3}) on the basis of a statistical method. Here we use the results of that work.

The process of formation of composite systems is determined by the hydrodynamic properties of liquid helium and by heat transfer in the normal phase, which, as experiment shows, has a convection-turbulent character. Turbulence must inevitably destroy and mix the boundary between the two phases, breaking it up into drops. But precisely this phenomenon, which in the case of contact between two ordinary liquids would lead to the complete disappearance of the boundary, here, on the contrary, serves as a necessary link in the mechanism of transfer of thermal energy between the two phases and explains the very existence of the visible boundary. In superfluid helium turbulence rapidly dies out, since the cause of its occurrence is absent: the temperature is rapidly equalized by the special quantum mechanism of thermal conductivity.

It was noted in (^{3}) that the linear dimensions of the composite systems directly participating in the phenomenon of multiple roton production must, generally speaking, lie between the roton wavelength $\lambda = h/p_0$ and the mean free path of an elementary excitation in the medium. It is not difficult to show that, in the experiment under consideration, the conditions for the formation of such systems are satisfied. Calculating the main characteristics of turbulent motion in the normal phase from the heat-transfer conditions, we find for the internal scale of turbulence the value $\lambda_0 \simeq 2 f(r_0/D)\, \mathrm{Re}_{\mathrm{cr}}^{3/4} \cdot 10^{-4}$ cm, where the function f, which depends on the linear dimensions of the vessel, is of order unity, while the critical value of the number Re cannot be very large for liquid helium. The fragmentation of the boundary may be considered on the basis of the works (^{4}, ^{5}). Taking from the data of work (^{5}) the critical Weber number, we obtain for the drop sizes the expression

$$\delta_0 \simeq \left(\frac{\sigma \cdot 10^{-7}}{3.7\nu^2\rho}\right)^{1/3} \frac{\mathrm{Re}_{\mathrm{cr}}}{\mathrm{Re}}\, D^{1/3}, \tag{2}$$

σ is the effective coefficient of surface tension ($\simeq 2.9 \cdot 10^{-3}$ erg/cm^2 at $\Delta T = 0.3$°K), ν is the kinematic viscosity of helium. Formula (2) gives the value $\delta_0 \simeq 7.7 \cdot 10^{-6}\,\mathrm{Re}_{\mathrm{cr}}$ cm, which satisfies the conditions indicated above.

4. In our approximation, to take into account the "background"of the medium we measure the energy in considering composite systems from E_0—the mean energy density at temperature T_0. We take the interactions between rotons into account by means of "effective" values of the parameters in the formula for the roton energy $\varepsilon(p) = \Delta + (p - p_0)^2/2\mu$.

If $\bar{\varepsilon}$ is the mean roton energy, ω is the frequency of formation of systems per 1 cm^2 of boundary surface per second, and $\bar{n}$ is the mean number of rotons produced in the decay of one system, then the heat flux through the boundary is $q = \bar{\varepsilon}\bar{n}\omega$. It is not difficult to see that the exponent α in formula (1) may be calculated from the relation

$$\alpha = \frac{d\ln(\bar{n}/\tau)}{d\ln(\bar{w}/\tau)}, \tag{3}$$

where

$$\frac{\bar{w}}{\tau} = \frac{E - E_0}{\Delta}\,\frac{4\pi}{3}\lambda^3, \qquad \tau = \left(\frac{R}{\lambda}\right)^3,$$

R is the "radius" of the composite system (notations of work (^{3})). In Fig. 1 the course of the curve of the quantity α^{-1}, co-

which is measured in experiment (^{1}). Near the λ-point the quantity w/τ must be approximately equal to unity, since in this case all degrees of freedom of the system are excited, and the "number" of excitations is comparable with the number of atoms. Setting $\ln(w/\tau) = 0$, we find $\alpha^{-1} = 1.91$. This theoretical value is in agreement with experiment (1.9 ± 0.1).

The coefficient a can be calculated only approximately. We have $\bar{n} = w\zeta(w/\tau)$ (see (^{3})); for $w/\tau = 1$, $\zeta = 0.25$; thus,

$$\bar{n} \approx 0.25\,(E - E_0)\delta_0^3/\Delta.$$

Fig. 1

Figure 1: Fig. 1

But the frequency ω is, in order of magnitude, u/δ_0^3, where u is the mean convection velocity (in our case $\approx$ 4 cm/sec). Taking $\varepsilon \sim \Delta$, we obtain $q \approx 0.25(E - E_0)u$. Further, it is obvious that $E - E_0 = c(T - T_0)$, where c, in our approximation, may be taken as the heat capacity of normal helium near T_λ (~ 0.9 cal/g $\cdot$ deg). For $\Delta T = 0.3$°K, hence, we find $q \approx 0.165$ W/cm^2, which is in satisfactory agreement with experiment.

Fig. 1

5. The idealization of the problem consisted in the fact that the properties of the liquid on the two sides of the boundary were regarded as fundamentally different, which cannot be exact near the λ-point. The results of the calculations show, however, that the theory correctly takes into account the basic mechanism of the phenomenon.

The author expresses sincere gratitude to V. P. Peshkov for repeated fruitful discussions of the problem.

Mathematical Institute named after V. A. Steklov
Academy of Sciences of the USSR

Received
22 V 1958

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