Mathematics

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Abstract

Full Text

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ASYMPTOTIC EXPANSION FOR NONHOMOGENEOUS MARKOV CHAINS

(Presented by Academician A. N. Kolmogorov, 20 VIII 1956)

A nonhomogeneous Markov chain is studied with a finite number $s > 1$ of possible states $e_1, \dots, e_s$ and transition probabilities $p_{\alpha\beta}^{(k)}$ from state e_α at the $(k-1)$-st step to state e_β at the k-th step.

Let the random variable $\zeta_n^{(\alpha)}$ denote the number of visits to state e_α during the first n steps. Then, for the probability $P_\gamma(m)$ that the random vector $\zeta_n = (\zeta_n^{(1)}, \dots, \zeta_n^{(s)})$ assumes the value $m = (m_1, \dots, m_s)$, under the condition that e_γ is the initial state, the following theorem is valid.

Theorem. *Suppose that the following conditions are satisfied:*

A. $p_{\alpha\beta}^{(l)} \geq \lambda p_{\alpha\beta}^{(k)}$ for all α, β, k, l, where the constant $\lambda > 0$.

B. The set of states of the chain forms one essential class.

C. The rank r of the chain is equal to s ([1]).

Then, for any γ and integer $k > 0$, uniformly in all $(m_1, \dots, m_s)$, we have

$$\sqrt{D_n^{(1)} \cdots D_n^{(s-1)}}\, \mathcal{P}_\gamma(m) = g_{s-1}(x) + \sum_{j=1}^{k} n^{-j/2} P_{\gamma j}\Big(-\frac{\partial}{\partial x} g(x)\Big) + O\Big(\frac{1}{n^{(k+1)/2}}\Big). \tag{1}$$

Here

$$g_{s-1}(x) = \frac{1}{\sqrt{(2\pi)^{s-1}\Delta_n}} \exp\left[-\frac{1}{2} Q_n^{-1}(x)\right]$$

is the density of the $(s-1)$-dimensional normal distribution,

$$x = \left(\frac{m_1 - E_n^{(1)}}{\sqrt{D_n^{(1)}}}, \dots, \frac{m_{s-1} - E_n^{(s-1)}}{\sqrt{D_n^{(s-1)}}}\right);$$

$E_n^{(\alpha)} \asymp n$, $D_n^{(\alpha)} \asymp n$; the quadratic form $Q_n(x) = Q_n(x_1, \dots, x_{s-1})$ is positive definite; Δ_n is the determinant of this form. $P_{\gamma j}(it)$ is a polynomial of degree not exceeding $3j$ in the components of the vector $it = (it_1, \dots, it_{s-1})$. The coefficients of the polynomial are real, depend on γ and n, but are uniformly bounded for all n. $P_{\gamma j}\big(-\frac{\partial}{\partial x} g_{s-1}(x)\big)$ means that, in place of the powers $-it_\alpha$, derivatives of $g_{s-1}(x)$ with respect to the corresponding component x_α are taken ([2]).

If condition C is violated, but $r > 1$, then in equality (1) s should be replaced by r.

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References

[1] V. A. Statulevičius, DAN, 107, No. 4 (1956).
[2] S. Kh. Sirazhdinov, *Limit Theorems for Homogeneous Markov Chains*, Tashkent, 1955.

Note: Figure translations are in progress. See original paper for figures.