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# E. A. Krasilshchikova

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**Abstract**

**Full Text**

**E. A. Krasilshchikova**

## **Unsteady Motions of a Wing of Finite Span in a Compressible Medium**

*(Presented by Academician L. I. Sedov, 13 VI 1957)*

### **Hydromechanics**

1. Spatial motions of a compressible fluid are investigated, caused by the unsteady motion of a wing of finite span within an unbounded volume of fluid at rest at infinity. We consider the motion of the wing under such conditions that it produces small disturbances. The problem is linearized and the generally accepted assumptions of thin-wing theory are made <sup>(1,2)</sup>. The solution is constructed in fixed coordinate axes  $xOyz$ , which determine the space of motion of the wing.

The law of motion of the wing is given in the form

$$x = F(t), \tag{1}$$

where  $F$  is an arbitrary continuous function of time.

The normal component of the velocity is given in the form

$$\begin{aligned} \Phi_n &= -F'(t)\beta(x, y, t) + A_1(x, y, t) = \\ &= A(x, y, t), \end{aligned} \tag{2}$$

where the function  $\beta$  (the angle of attack of the wing elements) and  $A_1$  are specified at every point of the wing surface and are integrable functions of their arguments. The first term corresponds to the basic motion of the wing, the second to small additional motions of the wing, in which the wing surface may deform. The velocity potential satisfies the three-dimensional wave equation

$$a^2\varphi_{xx} + a^2\varphi_{yy} + a^2\varphi_{zz} - \varphi_{tt} = 0.$$

**Fig. 1**

**Fig. 2**

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

2. We introduce into consideration a four-dimensional space determined by the coordinates  $x, y, z$ , and  $t$ , and formulate the boundary-value problem (3).

Find a function  $\varphi(x, y, z, t)$  that satisfies the wave equation, whose derivatives vanish at infinity, and which, in the space of the variables  $(x, y, t)$ , satisfies the following boundary conditions: in

of the region  $V$  (Fig. 1) the derivative  $\varphi_z = A(z, y, t)$ ; in the region  $V_1$  the derivative  $\varphi_t = 0$ ; in the region  $V_2$  the function  $\varphi = 0$ .

The region  $V$  is bounded by the surface  $\Sigma^*$ , which is the geometric locus of curves representing the laws of motion of the points of the wing contour. The region  $V_1$  is bounded by two planes tangent to the surface  $\Sigma^*$  along the curves  $AA'$  and  $BB'$ , and by a part of the surface  $\Sigma^*$  formed by curves representing the laws of motion of the points of the trailing edge of the wing. The curves  $AA'$  and  $BB'$ , respectively, represent the laws of motion of the points  $A$  and  $B$ , the extreme left and extreme right points of the wing contour. The region  $V_2$  is the part of the space  $(xyt)$  located outside the region  $V + V_1$ .

In Fig. 2 the plane region  $\Sigma$  represents the projection of the wing onto the plane  $xOy$  at a certain instant of time  $t_1$ .

3. Let us turn to solutions of the equation

$$(u_1^2 - a^2)\varphi_{1xx} - a^2\varphi_{1yy} - a^2\varphi_{1zz} + \varphi_{1tt} + 2u_1\varphi_{1tx} = 0 \quad (3)$$

of the form ((4), Chap. 1, § 3)

$$\varphi^*(x, y, z, t) = \frac{f\left\{\xi, \eta, t - \frac{u_1(x-\xi)}{u_1^2 - a^2} + \frac{a}{u_1^2 - a^2} \sqrt{(x-\xi)^2 - k^2(y-\eta)^2 - k^2z^2}\right\}}{\sqrt{(x-\xi)^2 - k^2(y-\eta)^2 - k^2z^2}} \quad (4)$$

$$\left(k^2 = \frac{u_1^2}{a^2} - 1\right),$$

where  $f$  is an arbitrary function of its arguments.

At each point  $M(\xi, \eta, 0)$  of the plane  $xOy$  we place sources with potentials  $\varphi^*$ .

Formula (4) shows that the variables

$$\xi, \quad \eta, \quad \tau = t - \frac{u_1(x-\xi)}{u_1^2 - a^2} + \frac{a}{u_1^2 - a^2} \sqrt{(x-\xi)^2 - k^2(y-\eta)^2 - k^2 z^2} \quad (5)$$

satisfy the equation

$$(u_1^2 - a^2)(t - \tau)^2 + (x - \xi)^2 + (y - \eta)^2 + z^2 - 2u_1(x - \xi)(t - \tau) = 0. \quad (6)$$

This equation is the equation of a surface in the space  $xyt$ , which can be obtained as the intersection of the four-dimensional characteristic conoid of equation (3) with the hyperplane  $\zeta = 0$ .

By virtue of the linearity of equation (3), its solution is the function

$$\varphi_1(x, y, z, t) = \iint \frac{f \left\{ \xi, \eta, t - \frac{u_1(x-\xi)}{u_1^2 - a^2} + \frac{a}{u_1^2 - a^2} \sqrt{(x-\xi)^2 - k^2(y-\eta)^2 - k^2 z^2} \right\}}{\sqrt{(x-\xi)^2 - k^2(y-\eta)^2 - k^2 z^2}} d\xi d\eta. \quad (7)$$

Introducing into (7) the new variables of integration  $\theta$  and  $\tau$ ,

$$\xi = x + u_1(t - \tau) - \sqrt{a^2(t - \tau)^2 - z^2} \sin \theta,$$

$$\eta = y - \sqrt{a^2(t - \tau)^2 - z^2} \cos \theta \quad (8)$$

and differentiating the resulting expression with respect to  $z$ , we arrive at the known relation ((4), Chap. I, § 3)

$$f(x, y, t) = -\frac{1}{2\pi} \varphi_{1z}(x, y, 0, t). \quad (9)$$

Introduce the element  $dS_1$  of the surface (6) and represent the solution (7) in the form

$$\varphi_1(x, y, z, t) = \frac{u_1^2 - a^2}{2\pi} \iint_{S_1(x, y, z, t)} \frac{\varphi_{1z} \left\{ \xi, \eta, t - \frac{u_1(x-\xi)}{u_1^2 - a^2} + \frac{a}{u_1^2 - a^2} r^* \right\}}{\sqrt{(u_1^2 - a^2)^2 r^{*2} + [a(x - \xi) - u_1 r^*]^2 + a^2 k^4 (y - \eta)^2}} dS_1 \quad (10)$$

$$(r^* = \sqrt{(x - \xi)^2 - k^2(y - \eta)^2 - k^2 z^2}),$$

where the region of integration  $S_1$  is the surface defined by equation (6).

Putting in (10) the constant  $u_1 = 0$ , we obtain the solution of the wave equation in the form

$$\varphi(x, y, z, t) = -\frac{a}{2\pi} \iint_{S(x, y, z, t)} \frac{\varphi_z \left\{ \xi, \eta, t - \frac{1}{a} \sqrt{(x - \xi)^2 + (y - \eta)^2 + z^2} \right\}}{\sqrt{(1 + a^2)(x - \xi)^2 + (1 + a^2)(y - \eta)^2 + a^2 z^2}} dS, \quad (11)$$

where the region of integration  $S$  is the surface of that branch of the hyperboloid extending to infinity

$$(x - \xi)^2 + (y - \eta)^2 + z^2 - a^2(t - \tau)^2 = 0, \quad (12)$$

which corresponds to values  $\tau < t$ .

4. We shall apply formula (11) to the boundary-value problem. This formula solves the problem effectively for such sets of variables  $x, y, z$ , and  $t$ , for which in the region of integration  $S$  the derivative  $\varphi_z$  is known everywhere.

In particular, by formula (11) one can compute the velocity potential everywhere on the surface of the wing when the wing has been moving for an indefinitely long time with supersonic speed and when the form of the wing in plan is such that the conditions ((<sup>4</sup>), Ch. I, § 3) are satisfied

$$\left| \frac{\Psi'(x_1)F'(t)}{\sqrt{\Psi'^2(x_1) + 1}} \right| > a, \quad (13)$$

$$\left| \frac{X'(x_1)F'(t)}{\sqrt{X'^2(x_1) + 1}} \right| > a, \quad (14)$$

respectively on the curves  $ACB$  and  $ADB$ , which form the contour of the wing and are given by the equations  $y_1 = \Psi(x_1)$  and  $y_1 = X_2(x_1)$  in moving coordinate axes (where  $y_1 = y$ ,  $x_1 = x - F(t)$ ), if in (11) one puts  $\varphi_z(\xi, \eta, \tau) = A(\xi, \eta, \tau)$ .

If the velocity of motion of the wing is supersonic and conditions (13) and (14) are satisfied, then for any point on the surface of the wing the region of integration  $S$  does not extend beyond the limits of the region  $V$ , where the derivative  $\varphi_z$  is prescribed (Fig. 3).

Fig. 3

In the general case, for an arbitrary set of variables  $x, y, z, t$ , in part of the region  $S$  the derivative  $\varphi_z$  proves to be unknown. In order that formula (11) correspond to the solution of the problem, it is necessary to determine  $\varphi_z$  everywhere in

the region of integration  $S$  from integral equations constructed on the basis of the boundary conditions of the problem in the region  $V_1$  or  $V_2$  of the space  $xyt$ .

In investigating various variants of unsteady motions of a wing, an essential role is played by the family of cones (12) for  $z = 0$ . These

cones play the same role as the families of straight lines  $X_1$  and  $X_2$  in constructing solutions of plane problems (<sup>3</sup>, p. 398). One of the cones  $\Omega$  of this family is shown in Fig. 1.

The results remain valid when the velocity of motion of the wing changes suddenly; when, in the course of the wing's motion, the type of additional unsteady motions of the wing changes repeatedly, with points of the wing surface possibly being included non-simultaneously in the additional motions; and when steady motion of the wing alternates with unsteady motion. In these cases, from the point of view of the form of the prescribed function  $A(x, y, t)$ , the region  $V$  is divided by the prescribed surfaces into a number of regions. The results also remain valid when the area of the wing surface changes to a finite value, which will affect the form of the prescribed boundary  $\Sigma^*$  of this region.

Institute of Mechanics  
Academy of Sciences of the USSR

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*Note: Figure translations are in progress. See original paper for figures.*

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