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Abstract

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DISPERSION RELATIONS FOR THE SCATTERING OF K-MESONS ON NUCLEONS

(Presented by Academician N. N. Bogolyubov, 15 V 1957)

PHYSICS

1. In the investigation of strong interactions, dispersion relations are so far the only method that gives, although not complete, nevertheless unconditional information, since the assumptions necessary for obtaining them essentially reduce to recognizing the existence of an S-matrix with the usual general properties and to the fulfillment of the strict causality condition (locality of the interaction). In this connection we shall analyze here the dispersion relations for the scattering of K-mesons on nucleons $K + N \to K + N$ (and $\widetilde{K} + N \to \widetilde{K} + N$), without dwelling on known details.

2. Regarding the K-meson as a scalar (Θ-meson) or pseudoscalar (τ-meson) particle, we associate with it the corresponding field $\varkappa_\rho(x)$. The index ρ enumerates four states linearly connected with the four particles $(K^+, K^0, \widetilde{K}^0, K^-)$, which differ in the projection of isotopic spin and in the sign of the hypercharge (or strangeness). In isotopic space these four fields form a double spinor, similarly to the four functions of the nucleon and antinucleon. We shall consider the interaction of K-mesons with nucleons of the Yukawa type $N \leftrightarrow K + Y,\ Y \leftrightarrow N + \widetilde{K}$. As will become clear below, other possible interactions will not explicitly be reflected in the dispersion relations.

Fig. 1

The amplitude of the scattering process $K + N \to K + N$ may be represented in the known way in the form

$$S_{\alpha\omega}(q, q') = \langle p's', q'\rho'|S|ps, q\rho\rangle =$$

$$= \frac{1}{(2\pi)^3\sqrt{4q^0{q'}^0}} \int e^{i(q'x - qy)} \left\langle p's' \left| \frac{\delta^2 S}{\delta\varkappa_\rho(x)\,\delta\varkappa_{\rho'}(y)} S^+ \right| ps \right\rangle dx\,dy, \qquad (1)$$

$$p^0 = \sqrt{\mathbf{p}^2 + M^2}, \dots, \quad {q'}^0 = \sqrt{{\mathbf{q}'}^2 + m^2},$$

where $\mathbf{p}, s, \mathbf{q}, \rho$ are the momentum and the other quantum numbers of the nucleon and meson before scattering, while the same symbols with primes are the quantum numbers after scattering; α and ω are composite indices of the initial and final states.

Using the equality

$$\frac{\delta^2 S}{\delta \varkappa_\rho(x)\, \delta \varkappa_{\rho'}(y)} S^+ = -\, i_\rho(x) i_{\rho'}(y) - i\, \frac{\delta i_\rho(x)}{\delta \varkappa_{\rho'}(y)}, \tag{2}$$

where $i_\rho(x) = i\, \dfrac{\delta S}{\delta \varkappa_\rho(x)} S^+$ is the current operator, we find that in the region of real momenta $p_i^2 > 0$ and positive energies $p_i^0 > 0$, connected with masses by the usual relation $p_i^2 = m_i^2$, the amplitude $S_{\alpha\omega}$ coincides with

$$H_{\alpha\omega}(q, q') = \frac{-i}{(2\pi)^3 \sqrt{4 q_0 q_0'}} \int dx\, dy\, e^{i(q'x - qy)} \left\langle p' s' \left| \frac{\delta i_\rho(y)}{\delta \varphi_\rho(x)} \right| ps \right\rangle. \tag{3}$$

The latter, by virtue of translational invariance, is represented in the form

$$H_{\alpha\omega}(q, q') = \frac{2\pi i}{\sqrt{4 q^0 {q'}^0}}\, \delta(p + q - p' - q')\, T^{ret}_{\alpha\omega}\left(\frac{q + q'}{2}\right). \tag{4}$$

If we introduce the accepted coordinate system $\mathbf{p} + \mathbf{p}' = 0$, then in it

$$\mathbf{q} = \mathbf{p} + \lambda \mathbf{e}, \qquad \mathbf{q}' = -\mathbf{p} + \lambda \mathbf{e} \quad (\mathbf{e}^2 = 1,\ (\mathbf{e} \cdot \mathbf{p}) = 0), \tag{5}$$

$$q^0 = {q'}^0 = \sqrt{m^2 + \mathbf{p}^2 + \lambda^2} = E, \qquad p^0 = {p'}^0 = \sqrt{\mathbf{p}^2 + M^2}, \qquad \lambda = \sqrt{E^2 - m^2 - \mathbf{p}^2},$$

and T^{ret} turns out to be a function of $E, \lambda\mathbf{e}$.

Having eliminated, by means of symmetrization, $\mathfrak{S}_+ f = f(\mathbf{e}) + f(-\mathbf{e})$, $\mathfrak{S}_- f = \dfrac{1}{\lambda}\{f(\mathbf{e}) - f(-\mathbf{e})\}$, the two-valuedness introduced by λ, one can analytically continue $\mathfrak{S}T^{ret}$ into the domain of complex values of E.

The rigorous proof of analyticity, constructed by N. N. Bogolyubov ([1]) for π-mesons, must be specially modified for this case, but we shall not do this here; rather, we postulate that the function $\mathfrak{S}T^{ret}(E)$ obtained as a result of analytic continuation has the necessary properties which permit one to write dispersion relations in the usual form, representing $\mathrm{Re}\, T^{ret}_{\alpha\omega} = D_{\alpha\omega}$ by means of an integral of $\mathrm{Im}\, T^{ret}_{\alpha\omega} = A_{\alpha\omega}$ along the entire real axis E.

To obtain relations connecting only physically observable quantities, we must reduce the interval of integration. Forming the combinations $(1 \pm P_{\rho\rho'})\mathfrak{S}T^{ret}_{\alpha\omega}$ (here $P_{\rho\rho'}$ is the operator interchanging the indices ρ and ρ'), which possess a definite parity with respect to E, we can get rid of the integration over negative energies. Finally, fixing the degree of growth of the amplitude as $E \to \infty$, with respect to which we shall assume that it grows linearly*, we obtain four dispersion relations for amplitudes with different symmetries, one of which we write here for illustration:

$$(1+P_{\rho\rho'})\mathfrak{S}_+\{D_{\alpha\omega}(E)-D_{\alpha\omega}(E_0)\} = \frac{2}{\pi}(E^2-E_0^2)P\int_0^\infty dE'\,\frac{E'\mathfrak{S}_+(1+P_{\rho\rho'})A_{\alpha\omega}(E')}{({E'}^2-E^2)({E'}^2-E_0^2)}.$$

3. But even after the exclusion of negative energies, the interval of integration contains the region $0 < E < \sqrt{m^2+\mathbf{p}^2}$, which does not correspond to scattering states and therefore cannot be determined from scattering experiments. To analyze this region, we use the representation for $A_{\alpha\omega}(E)$ that follows from (2) and from the assumption of the existence of a complete system of states

$$A_{\alpha\omega}(E,\mathbf{e}) = (2\pi)^4 i\sum_n \langle p's'|i^*_\rho(0)|\lambda\mathbf{e},n\rangle\langle\lambda\mathbf{e},n|i_{\rho'}(0)|ps\rangle$$

$$\cdot\,\delta\Big(\sqrt{M^2+\mathbf{p}^2}+E-\sqrt{M_n^2+\lambda^2}\Big) - (2\pi)^4 i\sum_n \langle ps'|i_{\rho'}(0)|-\lambda\mathbf{e},n\rangle$$

$$\cdot\,\langle-\lambda\mathbf{e},n|i_\rho(0)|ps\rangle\,\delta\Big(-\sqrt{M^2+\mathbf{p}^2}+E+\sqrt{M_n^2+\lambda^2}\Big).$$

* Such an assumption, under which the total cross section tends to a constant, seems quite natural. Such a si

The behavior of $A_{\alpha\omega}(E)$ is determined by the energy δ-functions entering into it. The arguments of the δ-functions vanish at the points

$$\pm E = \frac{\mathcal{M}^2-M^2-m^2-2\mathbf{p}^2}{2\sqrt{M^2+\mathbf{p}^2}}.$$

Thus determining is the mass spectrum of the possible intermediate states. By the laws of conservation of strangeness and nucleon charge the following states are allowed: a hyperon Y (Λ or Σ), a hyperon and a π-meson ($Y+\pi$), a nucleon and a K-meson $(N+K)^*$, etc.

The state Y belongs to the discrete spectrum; its contribution is easy to calculate, and it is proportional to the square of the NKY coupling constant. Thus, for example, the contribution of the Λ-hyperon, symbolically written, for pseudoscalar coupling has the form

$$\Gamma(\Lambda) = G^2 \left\{ \hat{\delta}_{s's} \frac{2\mathcal{M}(\mathbf{p}^2 + M^2) - (\mathcal{M}^2 + M^2 - m^2)M}{4(M^2 + \mathbf{p}^2)^{3/2}} - i\lambda_p \frac{(\sigma \cdot [\mathbf{p} \times \mathbf{e}])_{s's}}{2(M^2 + \mathbf{p}^2)} \right\} \delta(E - E_p),$$

but the continuous spectrum begins from the lowest two-particle state. Since the boundary of the continuous spectrum

$$E_p^{\Lambda+\pi} = \frac{(\mathcal{M} + \mu)^2 - M^2 - m^2 - 2\mathbf{p}^2}{2\sqrt{M^2 + \mathbf{p}^2}}$$

lies below the boundary of the scattering states $E_1 = \sqrt{m^2 + \mathbf{p}^2}$ even for the most favorable case of forward scattering (i.e. $\mathbf{p} = 0$ in our system), it is still necessary somehow to estimate the integral from $E_p^{\Lambda+\pi}$ to E_1. For this one may use the analytic expression for $A_{\alpha\omega}$ in the low-energy region ($E > m,\ E \sim m$) and then regard this same expression as valid in the region of purely imaginary momenta ($E < m$). Without discussing the legality of such a procedure, one may hope that it is suitable at least for a rough estimate of the contribution.

4. If one assumes any other interactions, for example the direct interaction of π-mesons with K-mesons according to the Goldhaber–Schwinger hypothesis ([2]), then it is clear that they should not be taken into account explicitly in the dispersion relations, since the lowest state for such an interaction will lie above the threshold of the scattering states and is taken into account automatically in the experimental $A_{\alpha\omega}(E)$ for $E > m$.

Thus, the magnitude of the contribution from the hyperon pole makes it possible to estimate the YKN coupling constant. Unfortunately, this result is somewhat blurred owing to the existence of an indeterminate contribution from the portion of the continuous spectrum.

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CITED LITERATURE

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* The first two states $Y,\ Y+\pi$ are allowed only in the process $\widetilde{K}+N \to \widetilde{K}+N$.

Note: Figure translations are in progress. See original paper for figures.