A MATRIX METHOD FOR THE ANALYSIS AND SYNTHESIS OF ELECTRONIC-PULSE AND RELAY-CONTACT (NONPRIMITIVE) CIRCUITS

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Abstract

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A MATRIX METHOD FOR THE ANALYSIS AND SYNTHESIS OF ELECTRONIC-PULSE AND RELAY-CONTACT (NONPRIMITIVE) CIRCUITS

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Various methods for studying relay-contact circuits have been proposed by M. A. Gavrilov ([1]), D. A. Huffman ([2]), and V. I. Shestakov ([3,4]). In the present note a matrix method is developed for the analysis and synthesis of so-called nonprimitive circuits, admitting realization both by relay-contact and by electronic-pulse circuits.

1. Let us consider an electrical circuit Q having $n+s$ input buses $x^1, \dots, x^{n+s}$ and $p+s$ output buses $f^1, \dots, f^{p+s}$, each of which at any fixed instant of time t ($t = 1, 2, \dots$) may be in one of two states, differing, depending on the method of physical realization of the circuit, by the presence or absence of a voltage pulse at the instant t, by one or another voltage level, by the opening or closing of the contacts of the relay controlled by the given bus, etc. To one of these states (the excited one) the number 1 is assigned, to the other the number 0, so that if, for example, at the instant t only the i-th input and the k-th output buses are excited, then $x_t^i = f_t^k = 1$ and $x_t^j = 0$ for $j \neq i$, $f_t^l = 0$ for $l \neq k$.

A circuit is called **primitive** if the set X_t of the quantities $x_t^1, \dots, x_t^{n+s}$, determining the states of the input buses of the circuit, completely determines the states of the output buses of the circuit, i.e. the set $F_t = (f_t^1, \dots, f_t^{p+s})$; in other words, if the relations

$$f_t^i = f^i(x_t^1, \dots, x_t^{n+s}), \qquad i = 1, 2, \dots, p+s. \tag{1}$$

hold.

The functions (1) can be realized both by contact circuits, as was first shown by V. I. Shestakov ([5]), and by electronic-pulse circuits (see, for example, ([6,7])).

Let us now introduce feedbacks with unit time delay, putting

$$x_{t+1}^{n+i} = f_t^{p+i}, \qquad i = 1, 2, \dots, s, \tag{2}$$

Fig. 1

Figure 1: Fig. 1

i.e. let us identify the state of the input bus x^{n+i} at the instant $t+1$ with the state of the output bus f^{p+i} at the instant t. Denote $x^{n+i}_{t+1} = \varphi^i_t$.

We shall call **nonprimitive** the circuit P obtained from the circuit Q by introducing feedbacks with delay (Fig. 1*). From the equations of the primitive circuit (1) and the feedbacks (2), the equations of the nonprimitive circuit can be obtained:

$$\begin{aligned} f^i_{t+1} &= f^i(x^1_{t+1}, \dots, x^n_{t+1}, \varphi^1_t, \dots, \varphi^s_t), \qquad i = 1, 2, \dots, p; \\ \varphi^j_{t+1} &= \varphi^j(x^1_{t+1}, \dots, x^n_{t+1}, \varphi^1_t, \dots, \varphi^s_t), \qquad j = 1, 2, \dots, s. \end{aligned} \tag{3}$$

* In Fig. 1 the delay lines are shown by shaded rectangles.

By equations (3), the input of the circuit at the instant $t+1$, i.e. the set $X_{t+1} = (x^1_{t+1}, \dots, x^n_{t+1})$, and the state of the circuit at the same instant, i.e. the set $\Phi_t = (\varphi^1_t, \dots, \varphi^s_t)$, completely determine the output of the circuit at the instant $t+1$ and the state of the circuit at the instant $t+2$, i.e. the sets $F_{t+1} = (f^1_{t+1}, \dots, f^p_{t+1})$ and $\Phi_{t+1} = (\varphi^1_{t+1}, \dots, \varphi^s_{t+1})$.

Let us note that circuit equations in which time delays that are multiples of the unit delay are used can be reduced to the form (3) by applying additional functions φ.

Fig. 1

The realization of feedbacks with time delay may be carried out by using electromagnetic or electronic relays with a unit operating time, various kinds of delay lines, triggers, etc.

Let us dwell in somewhat greater detail on a trigger circuit ([3]) controlled by pulses that either arrive or do not arrive simultaneously at both grids (or both cathodes). Its states satisfy the equation

$$\varphi_{t+1} = \overline{\varphi}_t y_{t+1} \vee \varphi_t \overline{y}_{t+1}, \tag{4}$$

where φ_t is the state function of the trigger at the instant $t+1$, equal to 1 if the left triode is locked, and equal to 0 if the left triode is open. The quantity y_{t+1} takes the value 1 when there is a pulse at the instant $t+1$ and the value 0 when there is no pulse; a bar over a letter denotes logical negation, and the sign $\vee$ denotes logical addition.

Relays and delay lines may be replaced by triggers with a unit operating time. Indeed, any of the equations of the second line of (3) assumes the form (4), if we put

$$y^i_{t+1} = \overline{\varphi}^i_t \varphi^i(x^1_{t+1}, \dots, x^n_{t+1}, \varphi^1_t, \dots, \varphi^{i-1}_t, 0, \varphi^{i+1}_t, \dots, \varphi^s_t) \vee$$

$$\vee\, \varphi^i_t \overline{\varphi}^i(x^1_{t+1}, \dots, x^n_{t+1}, \varphi^1_t, \dots, \varphi^{i-1}_t, 1, \varphi^{i+1}_t, \dots, \varphi^s_t).$$

The function y may be called the **switching function of the trigger**.

2. Let us call a **simple vector** the set $\widetilde{\Lambda} = (\tilde{\lambda}^0, \dots, \tilde{\lambda}^{2q-1})$, composed of $2q$ coordinates $\tilde{\lambda}^i$, exactly one of which is equal to 1, while the others are zeros. To each set $\Lambda = (\lambda^1, \dots, \lambda^q)$ of q quantities λ^i we assign the simple vector $\widetilde{\Lambda} = (\tilde{\lambda}^0, \dots, \tilde{\lambda}^{2q-1})$, in which the coordinate equal to 1 is the coordinate whose number i, when written in binary arithmetic, has the form $\lambda^q, \dots, \lambda^1$ $(i \sim \lambda^q, \dots, \lambda^1)$.*

Between the elements of the set λ^j and the coordinates of the simple vector $\tilde{\lambda}^i$ the following relations hold**:

$$\tilde{\lambda}^{\alpha_q, \dots, \alpha_1} = \prod_{j=1}^{q} [\lambda^j]^{\alpha_j}; \qquad \lambda^j = \bigvee_{\alpha_1, \dots, \alpha_q;\, \alpha_j = 1} \tilde{\lambda}^{\alpha_q, \dots, \alpha_1}. \tag{6}$$

In expression (6), as everywhere below, $[a]^b = ab \vee \overline{a}\,\overline{b}$, i.e. $[a]^b = 1$ when $a = b$, and $[a]^b = 0$ when $a \neq b$.

Let us assign to the state of the circuit at the instant $t + 2$, i.e. to the set $\Phi_{t+1} = (\varphi^1_{t+1}, \dots, \varphi^s_{t+1})$, the **simple vector of the state of the circuit** $\widetilde{\Phi}_{t+1} = (\widetilde{\varphi}^0_{t+1}, \dots, \widetilde{\varphi}^{2s-1}_{t+1})$. In this case each state of the circuit will be assigned a number —the number of the nonzero coordinate of the corresponding simple vector.

* We write the signs of binary numbers separated by commas so as not to confuse binary numbers with products. To distinguish binary numbers from sets, the latter are enclosed in parentheses.

** The sign $\prod$ denotes logical multiplication.

Using (6) and (3), it is not difficult to obtain a formula relating the coordinates of the simple vectors $\widetilde{\Phi}_{t+1}$ and $\widetilde{\Phi}_t$:

$$\widetilde{\varphi}^{\beta_s, \dots, \beta_1}_{t+1} = \bigvee_{\alpha_s, \dots, \alpha_1} \widetilde{\varphi}^{\alpha_s, \dots, \alpha_1}_t a_{\alpha_s, \dots, \alpha_1;\, \beta_s, \dots, \beta_1}(x^1_{t+1}, \dots, x^n_{t+1}); \qquad \widetilde{\Phi}_{t+1} = \widetilde{\Phi}_t A(X_{t+1}). \tag{7}$$

The **state matrix of a nonprimitive circuit**

$$A(X_{t+1}) = \left\| a_{\alpha_s, \dots, \alpha_1;\, \beta_s, \dots, \beta_1}(x^1_{t+1}, \dots, x^n_{t+1}) \right\|$$

is related to equations (3) by the formula

$$a_{\alpha_s,\dots,\alpha_1;\,\beta_s,\dots,\beta_1}(x^1_{t+1},\dots,x^n_{t+1}) = \prod_{i=1}^{s}\left[\varphi^i(x^1_{t+1},\dots,x^n_{t+1},\alpha_1,\dots,\alpha_s)\right]^{\beta_i}. \quad (8)$$

It follows directly from (8) that, for any fixed values of the quantities $x^1_{t+1},\dots,x^n_{t+1}$, each row of the matrix A contains exactly one element equal to one, while the remaining elements are zeros. We shall call such matrices **simple**; they form a semigroup under multiplication defined by the formula

$$C = AB, \qquad c_{\alpha_s,\dots,\alpha_1;\,\beta_s,\dots,\beta_1} = \bigvee_{\gamma_s,\dots,\gamma_1} a_{\alpha_s,\dots,\alpha_1;\,\gamma_s,\dots,\gamma_1} b_{\gamma_s,\dots,\gamma_1;\,\beta_s,\dots,\beta_1}. \quad (9)$$

From (8) there follows immediately the relation

$$\widetilde{\Phi}_t = \widetilde{\Phi}_0 A(X_1)A(X_2)\cdots A(X_t), \qquad X_i = (x^1_i,\dots,x^n_i) \quad (i = 1,\dots,t), \quad (10)$$

which relates the state of the circuit at the instant t to its initial state.

From the matrix $A(X)$, the second group of equations (3) of the nonprimitive circuit can be uniquely reconstructed:

$$\varphi^i(x^1_{t+1},\dots,x^n_{t+1},\varphi^1_t,\dots,\varphi^s_t) = \bigvee_{\substack{\alpha_1,\dots,\alpha_s\\ \beta_1,\dots,\beta_s;\,\beta_i=1}} a_{\alpha_s,\dots,\alpha_1;\,\beta_s,\dots,\beta_1}(x^1_{t+1},\dots,x^n_{t+1})[\varphi^1_t]^{\alpha_1}\cdots[\varphi^s_t]^{\alpha_s}. \quad (11)$$

Assigning to the input of the circuit $X_{t+1} = (x^1_{t+1},\dots,x^n_{t+1})$ and to its output $F_{t+1} = (f^1_{t+1},\dots,f^p_{t+1})$ the simple input and output vectors

$$\widetilde{X}_{t+1} = (\tilde{x}^0_{t+1},\dots,\tilde{x}^{2^n-1}_{t+1}) \quad \text{and} \quad \widetilde{F}_{t+1} = (\tilde{f}^0_{t+1},\dots,\tilde{f}^{2^p-1}_{t+1}),$$

we obtain a relation connecting their coordinates:

$$\tilde{f}^{\beta_p,\dots,\beta_1}_{t+1} = \bigvee_{\alpha_1,\dots,\alpha_n} \tilde{x}^{\alpha_n,\dots,\alpha_1}_{t+1} l_{\alpha_n,\dots,\alpha_1;\,\beta_p,\dots,\beta_1}(\varphi^1_t,\dots,\varphi^s_t); \qquad \widetilde{F}_{t+1} = \widetilde{X}_{t+1}L(\Phi_t); \quad (12)$$

the elements of the simple rectangular **reaction matrix** of the circuit

$$L(\Phi_t) = \left\| l_{\alpha_n,\dots,\alpha_1;\,\beta_p,\dots,\beta_1}(\varphi^1_t,\dots,\varphi^s_t) \right\|$$

are calculated from (3) by the formula

Fig. 2

Figure 2: Fig. 2

$$l_{\alpha_n,\dots,\alpha_1;\,\beta_p,\dots,\beta_1}(\varphi_t^1,\dots,\varphi_t^s) = \prod_{i=1}^{p} \left[f^i(\alpha_1,\dots,\alpha_n,\varphi_t^1,\dots,\varphi_t^s)\right]^{\beta_i}. \qquad (13)$$

Conversely, the matrix $L(\Phi_t)$ uniquely determines the first group of equations (3):

$$f^i(x_{t+1}^1,\dots,x_{t+1}^n,\varphi_t^1,\dots,\varphi_t^s) = \bigvee_{\substack{\alpha_1,\dots,\alpha_n \\ \beta_1,\dots,\beta_p;\,\beta_i=1}} l_{\alpha_n,\dots,\alpha_1;\,\beta_p,\dots,\beta_1}(\varphi_t^1,\dots,\varphi_t^s)[x_{t+1}^1]^{\alpha_1}\cdots[x_{t+1}^n]^{\alpha_n}. \qquad (14)$$

3. The state and reaction matrices of a nonprimitive circuit make it possible to describe its operation in time easily. We shall therefore call this **analysis**

of a **circuit**—the construction of matrices from given equations, and **synthesis** —the restoration of the equations of a circuit from given matrices.

Let us give examples of the analysis and synthesis of nonprimitive circuits. In Fig. 2 a relay-contact circuit is shown (the notation corresponds to that adopted in (1)), which is described by the equations

$$\varphi_{t+1}^1 = (\varphi_t^1 \vee x_{t+1})(\overline{\varphi_t^2} \vee \overline{x_{t+1}}), \quad \varphi_{t+1}^2 = (\varphi_t^1 \vee x_{t+1})(\varphi_t^2 \vee \overline{x_{t+1}}), \quad f = x_{t+1}\overline{\varphi_t^1}\varphi_t^2. \qquad (15)$$

Fig. 2

The matrices of states and reactions constructed from formulas (8) and (13) have the form

$$A(x) = \begin{pmatrix} \overline{x} & x & 0 & 0 \\ 0 & x & 0 & \overline{x} \\ \overline{x} & 0 & x & 0 \\ 0 & 0 & x & \overline{x} \end{pmatrix};$$

$$L(\varphi^1,\varphi^2) = \begin{pmatrix} 1 & 0 \\ \overline{\varphi^2} \vee \varphi^1\varphi^2\overline{\varphi^1} & \end{pmatrix}. \qquad (16)$$

From (16) it is clear that if initially both relays are disconnected, then excitation of the output follows upon a repeated pressing of the button; after two button presses the circuit returns to any initial state.

Let us now synthesize a circuit for differential binary conversion of order s, i.e., a nonprimitive circuit with s feedbacks and two input buses x, y, whose state is preserved for $x_{t+1} = y_{t+1}$ and passes to the next (previous) state upon excitation only of x_{t+1} (only of y_{t+1}); moreover, for the first (last) state the preceding (following) one is the last (first). The state matrix of this circuit has the form

$$A(x,y) = \begin{pmatrix} [x]^y & x\overline{y} & 0 & 0 & \dots & 0 & 0 & 0 & \overline{x}y \\ \overline{x}y & [x]^y & x\overline{y} & 0 & \dots & 0 & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \dots & 0 & \overline{x}y & [x]^y & x\overline{y} \\ x\overline{y} & 0 & 0 & 0 & \dots & 0 & 0 & \overline{x}y & [x]^y \end{pmatrix}.$$

Noting that if $k \sim \alpha_s, \dots, \alpha_1$, then

$$k+1 \sim \alpha_s^{\overline{\alpha_{s-1}} \vee \dots \vee \overline{\alpha_1}}, \dots, \alpha_2^{\overline{\alpha_1}} \overline{\alpha_1}$$

and

$$k-1 \sim \alpha_s^{\alpha_{s-1} \vee \dots \vee \alpha_1}, \dots, \alpha_2^{\alpha_1} \alpha_1,$$

and making use of (11) and (5), we obtain equations of type (3)

$$\varphi_{t+1}^i = \varphi_t^i \cdot \overline{\left(x_{t+1}\overline{y_{t+1}}\varphi_t^1 \dots \varphi_t^{i-1} \vee \overline{x_{t+1}}y_{t+1}\overline{\varphi_t^1} \dots \overline{\varphi_t^{i-1}}\right)} \vee$$

$$\vee \overline{\varphi_t^i} \cdot \left(x_{t+1}\overline{y_{t+1}}\varphi_t^1 \dots \varphi_t^{i-1} \vee \overline{x_{t+1}}y_{t+1}\overline{\varphi_t^1} \dots \overline{\varphi_t^{i-1}}\right).$$

These equations make it possible to realize the circuit—for example, on flip-flop cells and diode circuits of logical multiplication. The circuit of the corresponding device is not given here for lack of space.

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Note: Figure translations are in progress. See original paper for figures.