ON SURGE IN COMPRESSORS

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

HYDROMECHANICS

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ON SURGE IN COMPRESSORS

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1. The study of the causes and character of self-oscillations of air flow rate and pressure in systems containing fans and compressors is of considerable theoretical and practical interest. A number of theoretical and experimental works have been devoted to investigating the problems of the onset of surge oscillations, determining their frequencies and intensities, and the influence of various factors on their character ([1–6]).

Before the work ([3])—a purely experimental one—it was believed that surge must always arise in those cases when the point of equilibrium operation lies on the rising portion of the characteristic. In work ([4]) an attempt was made to study the phenomenon theoretically within the framework of linear theory. However, according to the calculations of the authors of that work themselves, the inaccuracy of the formulas obtained led to a discrepancy between the calculated results and the experimental ones by a factor of 40 in one case and by a factor of 150 in another.

As in the works mentioned, so also in other works, there is no theoretical explanation of cases known from practice of the hysteretic character of surge; nor is there any explanation of a number of other characteristic features of surge, and the influence of external conditions is not considered.

Fig. 1

In the author's work ([6]) a theory of surge is set forth which takes into account the nonlinear features of the phenomenon. The theory developed makes it possible to explain the main phenomena occurring during surge, to indicate the reasons for differences in the character of surge oscillations, and to evaluate the influence of various factors on the region and character of surge. The theory makes it possible to predict new phenomena. In particular, it is shown in it that surge may also occur at regimes corresponding to the descending branches of the compressor characteristic, which before this work had been considered stable.

2. In the present note the question of the character of the excitation of surge

Fig. 2

Figure 2: Fig. 2

oscillations and of a method for suppressing them will be considered.

Consider a system containing a fan 2 (see Fig. 1), a suction pipeline 1, and a delivery pipeline 3, at the outlet of which there is a variable resistance in the form of a throttle 4.

Let us denote by p_0 the absolute pressure, by ρ_0 the density, by c_0 the speed of sound, and by Q_0 the volumetric flow rate in the suction pipeline. The corresponding quantities in the delivery pipeline will be denoted by p_k, ρ_k, c_k. Let the fan characteristic be given by the relation

$$\frac{p_k}{p_{01}} = \pi_1(Q_0), \tag{1}$$

where p_{01} is the static pressure before the fan. A typical characteristic is shown in Fig. 2, 1.

The throttle (network) characteristic

$$p_d - p_0 = \varphi(Q_R) \tag{2}$$

relates the magnitude of the excess pressure p ahead of the throttle to the volumetric flow rate Q_R through the throttle. In our case it may be regarded as a quadratic parabola (Fig. 2, 2).

We shall make the following assumptions: 1) the complex distributed acoustic system can be replaced by a system with one degree of freedom; 2) the pressure changes in the oscillatory regime are small.

Fig. 2

Then, as follows from (6), the motions in the system can be described by a system of two first-order differential equations:

$$\frac{dQ_0}{dt} = \frac{1}{L_a}\,[F(Q_0) - p], \tag{3}$$

$$\frac{dp}{dt} = \frac{1}{C_a}\,[Q_0 - \varphi_1(p)], \tag{4}$$

where $\varphi_1(p)$ is the inverse function of $\varphi(Q_R)$; C_a is the acoustic compliance of the system; L_a is the acoustic mass; $F(Q_0) = \pi_1(Q_0) - p_0$.

Eliminating time from equations (3) and (4), we obtain the differential equation of the integral curves:

$$\frac{dp}{dQ_0} = \frac{Q_0 - \varphi_1(p)}{F(Q_0) - p}\,\frac{L_a}{C_a}. \tag{5}$$

Let Q_0^*, p^* be the values corresponding to the equilibrium regime and serving as roots of the system of equations

$$Q_0 - \varphi_1(p) = 0, \qquad F(P_0) - p = 0.$$

Move the origin of coordinates to the singular point Q_0^*, p^*. Setting $Q_0 = Q_0^* + x$, $p = p^* + y$, we obtain in the new coordinates

$$\frac{dx}{dt} = \frac{1}{L_a}\left[F'x + \frac{F''}{2}x^2 + \frac{F'''}{6}x^3 + \cdots - y\right], \tag{6a}$$

$$\frac{dy}{dt} = \frac{1}{C_a}\left[x - \varphi_1' y - \frac{1}{2}\varphi_1'' y^2 - \frac{1}{6}\varphi_1''' y^3 - \cdots\right]. \tag{6b}$$

Here the derivatives F', F'', F''', φ_1', φ_1'', φ_1''', $\cdots$are taken for the values Q_0^*, p^*.

The equations of the first approximation will have the form

$$\frac{dx}{dt} = \frac{F'}{L_a}x - \frac{1}{L_a}y, \qquad \frac{dy}{dt} = \frac{1}{C_a}x - \frac{\varphi_1'}{C_a}y.$$

Then the condition for static stability of the regime will be

$$k > F'(Q_0^*), \tag{7a}$$

and the condition for self-excitation

$$\frac{dF(Q_0^*)}{dQ_0^*} > \frac{L_a}{kC_a}, \tag{7b}$$

where $k = 1/\varphi_1'$ is the tangent of the angle of inclination of the tangent to the network characteristic

at the operating point; $dF(Q_0)/dQ_0^*$ is the tangent of the angle of inclination of the tangent to the compressor characteristic at the operating point.

Approximately one may take

$$L_a = \rho_0 \left(\frac{l_1 \pi}{s_1} + \frac{0.5 l_2}{s_2} \right), \qquad C_a = \frac{0.5 l_2 s_2}{\rho_0 c_k^2}, \qquad c_k^2 = c_0^2 F(Q_0)^{\frac{\gamma-1}{\gamma}},$$

where l_1, s_1, l_2, s_2 are, respectively, the length and cross-sectional area of the suction and discharge pipelines, and γ is the adiabatic exponent.

The angular frequency ω at the moment the oscillations arise is determined by the expression $\omega = [(k - F'(Q_0^*))/kC_a L_a]^{1/2}$.

3. Let us proceed to consider the question of the character of excitation of surge oscillations. In this case one cannot confine oneself to considering the first-approximation equations; it is necessary to take into account the nonlinear terms in the expansion of the functions F and φ_1.

According to Lyapunov ([8,9]), the hard or soft character of the excitation is determined by the sign of a certain quantity g. In our case this Lyapunov coefficient will have the form

$$g = \frac{\pi L_a^2 C_a \sqrt{L_a C_a}}{8(1 - F'\varphi_1')\sqrt{1 - F'\varphi_1'}} \left\{ -\frac{F'}{L_a C_a^3} \varphi_1'^2 + \frac{F' F''^2}{L_a^4} - \left(\frac{F'^2}{L_a^2} - \frac{1}{L_a C_a} \right) \left[-\frac{\varphi_1'''}{C_a^2} + \frac{F'''}{L_a^2} \right] \right\} \tag{8}$$

and, under the condition that $-\left(\frac{F'^2}{L_a} - \frac{1}{L_a C_a} \right) > 0$. According to Lyapunov, if $g > 0$, then the excitation is hard; if $g < 0$, then the excitation is soft.

Let us suppose first that the network characteristic is linear in the neighborhood of the operating point of the characteristic; then $\varphi_1'' = \varphi_1''' = 0$. If the operating point lies near the maximum of the compressor characteristic (Fig. 2, 2), then $F' \cong 0$, and the sign of the coefficient g coincides with the sign of $F'''(Q_0^*)$. If

$$F''' < 0, \tag{9a}$$

then the excitation is soft; whereas if

$$F''' > 0, \tag{9b}$$

then the excitation is hard.

In passing to the ascending portion of the characteristic, the influence of the term $F' F''^2$ becomes apparent. Since $F' > 0$, then, when moving away from the point of maximum of the characteristic to the left, in order for the excitation to be hard, it is necessary to satisfy the condition

Fig. 3

Figure 3: Fig. 3

$$F'''(Q_0^*) > \frac{F' F''^2 C_a}{F'^2 C_a - L_a}. \tag{9c}$$

If the network characteristic is not rectilinear in the neighborhood of the operating point, then it is necessary to take into account the terms with φ_1'' and φ_1'''. For a quadratic dependence between pressure and flow rate, $p = \alpha Q^2$, we shall have $\varphi_1'' = -1/4p\sqrt{p\alpha}$, $\varphi_1''' = 3/8p^2\sqrt{\alpha p}$.

The results obtained show that surge is possible also on the descending branch of the characteristic in the neighborhood of the maximum point, if condition (8) is satisfied. In this case the surge proves to be hard, i.e., intense oscillations–the most dangerous ones–arise at once. In this case the onset and disappearance of surge will have a hysteresis character.

4. Let us consider the possibility of acting upon surge. We introduce feedback in such a way that the flow area s of the outlet throttle will be $s = s_0 + a\dot{p}$. Let

$$p_g - p_0 = \frac{Q}{bs}. \tag{2a}$$

Then equations (3) and (4), in the coordinates x, y, will take the form

$$L_a \dot{x} = F'x - y, \tag{3a}$$

$$(C_a + bpa)\dot{y} = x - bs_0 y. \tag{4a}$$

The stability conditions will be

$$F' < \frac{bs_0}{C_a + aps}; \qquad F' < \frac{1}{bt_0}.$$

Obviously, by the choice of the constant a one can always ensure a stable regime not only in the small, but also in the large (see ([6]), pp. 33-37).

Fig. 3

5. We now indicate a method for integrating the equation of compressor surge directly from the characteristics of the fan and of the network, under the assumption $L_a = \text{const}$, $C_a = \text{const}$ (i.e., $F = F(Q_0)$).

Fig. 4

Figure 4: Fig. 4

In the coordinate system $p_k - p_0$, Q_k (Fig. 2) we shall depict the characteristic of the compressor 1 and of the network 2. We shall regard Fig. 2 as the phase plane of equation (5). Then this equation receives a very simple geometrical interpretation.

Let the point M_1, with coordinates Q_1 and p_0, represent the state of the system. Change by L_a/C_a times the length of the segment $M_1A_1 = Q_k - \varphi_1(p)$, laying it off from the point M_1. Let this be the segment M_1A_3. Drawing from the point A_4, as from a center, an element of an arc of radius A_4M_1 through the point M_1, we obtain the points M_3 and M_2. Continuing the indicated construction, we successively construct the integral curve (for details see ([6])). Figure 3 shows the phase plane constructed by the indicated method. On it there is an unstable focus, two stable limit cycles, and one unstable one. Consequently, the system has hard surge of two different intensities. An analogous construction is performed in Fig. 4.

Fig. 4

Comparison of the theoretical results of the work with the experimental results showed very good agreement.

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Note: Figure translations are in progress. See original paper for figures.