HYDROMECHANICS

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Abstract

Full Text

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ON THE DYNAMIC EXPLOSION OF EQUILIBRIUM

Let us consider the equilibrium and unsteady motion of a perfect gas, taking into account internal gravitational forces.

We shall construct an example of an exact solution of the nonlinear equations of motion and equations of equilibrium in such a way that the distribution of density and pressure in a gas mass at rest in equilibrium represents the initial state for an unsteady motion of a gas of explosive type, arising and developing without the release of energy in the gas mass at rest.

It is possible that the phenomenon described below may be regarded as a phenomenon of a special instability of mechanical nature, which may be used to explain certain effects observed in variable stars.

We shall take the equations of one-dimensional unsteady gas motion with spherical symmetry in the form

$$\frac{\partial M}{\partial r} = 4\pi r^2 \rho;$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial r} + \frac{2\rho v}{r} = 0;$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{fM}{r^2} = 0; \tag{1}$$

$$\frac{\partial (p/\rho^\gamma)}{\partial t} + v\frac{\partial (p/\rho^\gamma)}{\partial r} = 0.$$

In the equations written above the notation is standard and clear from the equations themselves. We note only that f is the gravitational constant, and γ is an abstract constant.

Consider a gas at rest in which the state characteristics are distributed according to the laws:

$$v = 0; \qquad M_1 = \frac{20}{3}\pi A r^{0.6}; \qquad \rho_1 = \frac{A}{r^{2.4}}; \qquad p_1 = \frac{50\pi f A^2}{21\, r^{2.8}}. \tag{2}$$

Formulas (2) determine a solution of equations (1) for any positive value of the constant A. One may consider a gas sphere at rest in a vacuum with finite mass, for which, when $r < r^*$, formulas (2) are valid, while for $r > r^*$ the laws of distribution of the state characteristics are different. The equilibrium laws (2) also satisfy the equations of energy balance and radiation transfer for certain special properties of the coefficients of absorption and energy release*.

At the front of a shock wave propagating with speed c through the gas at rest, the following equalities must hold for M, ρ, v, and p[1]:

* For a model with a point energy source at the center of a star, it is sufficient[1] that the absorption coefficient χ be determined by a formula of the form $\chi = B\rho^w T^\nu$, where T is the temperature, and moreover

$$\frac{9-2\nu}{3-w-\nu} = 2.4.$$

In particular, if $\nu = -3.5$, then $w = 5/6$, which gives a formula close to Kramers' formula.

$$\begin{aligned} M_2 &= M_1; \\ \rho_2 &= \frac{\gamma+1}{\gamma-1}\rho_1\left(1+\frac{2}{\gamma-1}\frac{a_1^2}{c^2}\right)^{-1}; \\ v_2 &= \frac{2}{\gamma+1}c\left(1-\frac{a_1^2}{c^2}\right); \\ p_2 &= \frac{2\gamma}{\gamma+1}p_1\frac{c^2}{a_1^2}\left(1-\frac{\gamma-1}{2\gamma}\frac{a_1^2}{c^2}\right), \end{aligned} \tag{3}$$

where $a_1^2 = \gamma p_1/\rho_1$.

It is easy to verify that the following simple formulas determine a particular exact solution of equations (1):

$$M = \frac{2}{9}\frac{r^3}{ft^2}; \qquad \rho = \frac{1}{6\pi f t^2}; \qquad v = \frac{2}{3}\frac{r}{t}; \qquad p = \frac{K}{f}\left(\frac{1}{t^2}\right)^\gamma, \tag{4}$$

where K is an arbitrary positive constant.

Let us show that a gas at rest can pass into the motion (4) through a shock wave. Indeed, the relations at the shock wave are satisfied after substituting the equalities (4) into the left-hand sides, and the equalities (2) into the right-hand sides of conditions (3), if

$$r_2 = (30\pi f A t^2)^{5/12}; \qquad \gamma = \frac{7}{6}; \qquad K = \frac{4}{189\pi}(30\pi f A)^{5/6}, \tag{5}$$

where r_2 is the radius of the shock wave. Equality (5) determines, in particular, the law of motion of the shock wave. It is obvious that the total energy of the moving gas behind the shock front is equal to the initial energy for these same gas particles; therefore there are no additional sources of mechanical or thermal energy causing the motion under consideration.

Solution (4) is a special case of the more general solution (2), in which some constants are equal to zero and which gives nonlinear pulsational motions of a gas sphere; thus the obtained motion with complete monotonic expansion at decreasing velocities is a special case of pulsational motions characteristic of Cepheids.

The simple solution found is essentially connected with the equality $\gamma = 7/6$. However, the example considered suggests the possibility that analogous effects may arise under the action of small perturbations for other density distributions and, correspondingly, other values of γ.*

In the proposed note we draw attention to the new explosive type of instability of mechanical nature described above. It is easy to show that this instability is not connected with the presence of a singularity at the center of symmetry. Other examples of such instability in the absence of gravitational forces can also be indicated.

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CITED LITERATURE

[1] L. I. Sedov, *Methods of Similarity and Dimension in Mechanics*, 1954. [2] S. Rosseland, *The Theory of Pulsating Variable Stars*, IL, 1952.

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* The value of γ entering equation (1) may be taken equal to the polytropic exponent, not equal to the value c_p/c_v from condition (3). Then, proceeding from (4), one can construct a family of analogous solutions.

Note: Figure translations are in progress. See original paper for figures.