ON A GENERALIZATION OF THE THEORY OF ACCUMULATION OF DEVIATIONS AND ITS APPLICATION TO THE DETERMINATION OF SELF-OSCILLATIONS IN GENERATORS

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Abstract

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ON A GENERALIZATION OF THE THEORY OF ACCUMULATION OF DEVIATIONS AND ITS APPLICATION TO THE DETERMINATION OF SELF-OSCILLATIONS IN GENERATORS

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In the general aspect of the dynamic accuracy of automatic-control systems, the theory of the accumulation of deviations in linear dynamic systems under disturbances bounded in modulus acquires essential significance.

In the works of B. V. Bulgakov ([1]) and others ([2–5]) it was shown that, if the external forces are finite, then a damped linear system with constant parameters cannot acquire unbounded forced oscillations. However, under unfavorable conditions disturbances may accumulate and become significant. B. V. Bulgakov determined the upper limits of the deviations of a system under the assumption that the external forces $f_\rho(t)$ are connected by the conditions

$$|f_\rho(t)| \leq l \quad (\rho = 1, 2, \ldots, r),$$

and otherwise may be arbitrary single-valued and continuous functions of time t having a sufficient number of derivatives. The functions $f_\rho(t)$ may have discontinuities of the first kind, but their number in any finite interval of time t must be finite.

At the present time, in the theory of automatic control, problems arise of determining the accumulation of velocity, accelerations, integrals, and more complicated functions of the controlled quantity. The purpose of the present note is to calculate the accumulated deviations of complicated functions of the controlled quantity under the assumption of a bound on the disturbance $f(t)$ in modulus or a bound on the modulus of the derivative of the disturbance $\dot{f}(t)$, and also to develop a graphical-analytical method for constructing a diagram of the accumulation of deviations in time.

Let a stable linear automatic-control system be described by the equation

$$\Phi(s)L\{x(t)\} = g(s)L\{f(t)\}, \tag{1}$$

where $\Phi(s)$ and $g(s)$ are polynomials in s (s is the Laplace operator), and the degree m of the polynomial $\Phi(s)$ is higher than the degree n of the polynomial $g(s)$; $L\{x(t)\}$ and $L\{f(t)\}$ are the Laplace transforms, respectively, of $x(t)$ and $f(t)$.

Consider the question of the accumulation of functions of the controlled quantity of the form $X(t)$:

$$X(t) = a_0 x(t) + a_1 \dot{x}(t) + \dots + a_k x^{(k)}(t) +$$

$$+ b_0 \int_0^t x(t)\,dt + \dots + \underbrace{\int_0^t \dots \int_0^t}_{r} x\,dt, \quad k < m - n, \tag{2}$$

under the action on system (1) of a disturbance $f(t)$ bounded in absolute value: $f(t) \le l$.

According to (1), the Laplace transform of $X(t)$ is equal to:

$$L[X(t)] = \left[a_0 \frac{g(s)}{\Phi(s)} + a_1 s \frac{g(s)}{\Phi(s)} + \dots + a_{ks}^k \frac{g(s)}{\Phi(s)} + \frac{b_0}{s} \frac{g(s)}{\Phi(s)} + \dots + \frac{b_r}{s^r} \frac{g(s)}{\Phi(s)}\right] \cdot f(s). \tag{3}$$

Using the theorem on the convolution of functions, we obtain the expression for $X(t)$:

$$X(t) = \int_0^t \dot{H}(\tau) \cdot f(t-\tau)\,d\tau, \tag{4}$$

where

$$\dot{H}(t) = L^{-1}[H(s)] = \left[a_0 \frac{g(s)}{\Phi(s)} + \dots + a_{ks}^k \frac{g(s)}{\Phi(s)} + \frac{b_0}{s} \frac{g(s)}{\Phi(s)} + \dots + \frac{b_r}{s^r} \frac{g(s)}{\Phi(s)}\right] \cdot \frac{1}{s} \tag{5}$$

is the conditional "total transient" function of the system.

The accumulation of the function $X(t)$ under the restriction on the absolute value of $f(t)$ is determined by the conditions

$$|f(t-\tau)| = l, \qquad \operatorname{sign} \dot{H}(t) = \operatorname{sign} f(t-\tau) \tag{6}$$

over the entire time interval 0—t. Applying the accumulation condition for the function $X(t)$, we find for each t a sequence of instants of time $t_1, t_2, \dots, t_n$ that are roots of the equation $\dot{H}(t) = 0$. Then the maximum accumulated value of the function $X(t)$ will be equal to

$$X_{\max\max} = \lim_{t\to\infty} l\{[H(t_1)-H(0)]-[H(t_2)-H(t_1)]+[H(t_3)-H(t_2)]-\cdots\}, \quad (6a)$$

where $H(t_i)$ are the extrema of the "total transient" function $H(t)$.

The construction of the diagram of accumulation of deviations $X_{\max}(t)$ is easily carried out from the graph of the "total transient" function $H(t)$. The diagram of accumulation of deviations is constructed from segments of the conditional "total transient" function $H(t)$, enlarged by L times, for $\dot{H}(t) > 0$, and from mirror-reflected segments of $H(t)$ for $\dot{H}(t) < 0$, successively fitting each segment of the "transient process" at the instants of time $t_1, t_2, \dots, t_n$.

The accumulation diagram $X_{\max}(t)$ is a monotone curve with points corresponding to the instants of time $t_1, t_2, \dots, t_n$, at which the derivative of the accumulation diagram is equal to zero. The diagram of accumulation of deviations tends to the value $X_{\max\max}$.

In the case where the disturbance $f(t)$ is bounded in absolute value by the derivative, in formula (6a) the quantities $H(t_i)$ will be replaced by the values of the extrema of the corresponding conditional "total impulse" functions of system (1).

Let us establish the connection between self-oscillations and the process of accumulation of deviations in ⊃-generators ([6—20]), and let us determine the amplitude and frequency of the arising oscillations on the basis of an analysis of the process of accumulation of deviations.

Let the oscillatory circuit of an n-th order generator be under the action of a pulse arriving from an amplifier with positive feedback. The equation of the generator is

$$D(s) \cdot L\{\varphi(t)\} = L\{f[\dot{\varphi}(t)]\}, \quad (7)$$

where s is the Laplace operator; $f(\dot{\varphi})$ is the nonlinear characteristic of the amplifier.

$\varphi(t)$ can be determined on the basis of Duhamel' s integral

$$\varphi(t) = \int_0^t \dot{h}(\tau) f_1(t-\tau)\, d\tau, \quad (8)$$

where $h(t)$ is the transient function of the oscillatory circuit; $f_1(t)$ is the action on the oscillatory circuit.

Let us consider the process of accumulation of deviations $\varphi(t)$ in the oscillatory circuit of the generator.

The process of maximum accumulation of deviations $\varphi(t)$ on the interval $0 - t$ can be determined on the basis of the following expression for the magnitude of the accumulated deviation ([6–8,12]):

$$\varphi_{\max\max} = l\{h(t_1) - [h(t_2) - h(t_1)] + [h(t_3) - h(t_2)] - \cdots\},$$

where $h(t_i)$ are the extrema of the function $h(t)$, l is the modulus of the pulse.

At the moment when system (7) reaches the limiting maximum deviation, the derivative

$$\frac{d\varphi_{\max\max}}{dt} = 0.$$

Thus, the steady state of the system will be characterized by the fact that the last in the series of pulses is a pulse changing its sign when the sign of the velocity $\dot{\varphi}$ changes.

Noting that the establishment of self-oscillation in relay generators is described by the equations

$$D(s) \cdot L[\varphi(t)] = +L, \qquad \dot{\varphi} > 0;$$

$$D(s) \cdot L[\varphi(t)] = 0, \qquad \dot{\varphi} < 0,$$

the limiting state of the system can formally be obtained as its steady state under the influence of pulses of height $L/2$, synchronous with the sign of the velocity $\dot{\varphi}$.

Thus, the limiting cycle characterizing the self-oscillations of the relay generator (1) corresponds to the steady process arising in the accumulation of deviations in the same oscillatory circuit of the generator when it is acted upon by a disturbance $\bar{f}(t)$ bounded in modulus: $|\bar{f}(t)| = L/2$.

It is easy to see that the maximum deviation $\varphi_{\max\max}$ in generators arises in the case of a relay characteristic of the controlled amplifier.

Let us determine the period T and the amplitude A of the self-oscillation. The limiting deviation in the process of accumulation of deviations is

$$^1/_2\, \varphi_{\max\max} = {}^1/_2\, l\{h(t_1) - [h(t_2) - h(t_1)] + [h(t_3) - h(t_2)] - \cdots\},$$

therefore,

$$A = {}^{1}/_{2}\,\varphi_{\max\max}. \qquad (9)$$

Consideration of the time interval $t_{i+1} - t_i$, during which the sign of the pulse in the process of accumulation of deviations does not change, shows that for both a finite and an infinite time interval t, the last in the series of indicated pulses is a pulse of duration $0 - t_1$. Consequently, in the steady state the generator oscillations occur with period $T = 2t_1$, where t_1 is the time at which the circuit reaches the maximum value (for automatic-control systems, the overshoot value) in the transient process.

It is evident that the general conclusion concerning the commonality of self-oscillations and the limiting steady state of oscillations of the system in the process of accumulation of deviations remains valid also for multiperiodic oscilla-

than (19), since both limiting states are determined by analogous equations.

Generators with an "L"-shaped characteristic are examples of physical and technical systems in which motions with the greatest possible accumulated deviation are achieved (and realized).

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Note: Figure translations are in progress. See original paper for figures.