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Abstract

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MATHEMATICAL PHYSICS

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HEAT WAVES IN A THIN UNBOUNDED PLATE LYING ON A CONTINUOUS HOMOGENEOUS FOUNDATION

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In this note we consider problems on heat waves in a thin plate, whose thickness h is small, and therefore it is assumed that the temperature does not vary through the thickness; the specific heat c, density ρ, and thermal-conductivity coefficient λ are constant; the coordinate axes x, y are placed in the middle plane. Suppose that boundary conditions of the third kind are prescribed on the upper plane, while the lower plane is in close contact with a continuous and homogeneous foundation in planes parallel to Oxy. This problem has much in common with problems on the bending of plates on a continuous elastic foundation ([1]) and is closely connected with problems of the theory of heat conduction considered in ([2,3]).

We first consider an auxiliary problem. Let heat sources $q_3 = A_1 \sin\omega t \sin\alpha x \sin\beta y + A_2 \cos\omega t \sin\alpha x \sin\beta y$ be applied to the unbounded plate. We denote the heat flux leaving through the upper plane by q_1, and through the lower plane by $q_2 = (A_5 \sin\omega t + A_6 \cos\omega t)\sin\alpha x \sin\beta y$. The temperature of the plate is $T = (A_3 \sin\omega t + A_4 \cos\omega t)\sin\alpha x \sin\beta y$, $q_1 = \delta T$. Substituting their expressions for q_1, q_2, q_3, T into the heat-conduction equation and equating the coefficients of $\sin\omega t$ and $\cos\omega t$, we obtain

$$\begin{aligned} -\lambda h(\alpha^2+\beta^2)A_3 &= -c\omega A_4 - A_1 + \delta A_3 + A_5, \\ -\lambda h(\alpha^2+\beta^2)A_4 &= c\omega A_3 - A_2 + \delta A_4 + A_6. \end{aligned} \tag{1}$$

Between the coefficients A_5, A_6 and A_3, A_4 there exist relations determined by the properties of the homogeneous continuous foundation.

Suppose that the properties of the foundation are described by kernels $K_1(r), K_2(r)$, such that, under the action of a unit source varying according to the law $\sin\omega t$, the temperature of the upper surface of the foundation varies according to the law $T_1 = K_1(r)\sin\omega t + K_2(r)\cos\omega t$. Therefore, if we have a source $\cos\omega t$, then the temperature of the upper surface of the foundation varies according to the law

$$T_1 = K_1(r)\cos\omega t - K_2(r)\sin\omega t.$$

From this it is easy to determine the dependences between A_3, A_4 and A_5, A_6. Indeed, if the sources $A_5 \sin\omega t \sin\alpha x \sin\beta y$ act on the foundation, then

$$T^{(1)} = P_1(\alpha,\beta)\sin\omega t + P_2(\alpha,\beta)\cos\omega t, \tag{2}$$

where

$$\begin{aligned} P_1(\alpha,\beta) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K_1\left(\sqrt{x^2+y^2}\right)\sin\alpha x\sin\beta y\,dx\,dy,\\ P_2(\alpha,\beta) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} K_2\left(\sqrt{x^2+y^2}\right)\sin\alpha x\sin\beta y\,dx\,dy; \end{aligned} \tag{3}$$

in our case $P_1(\alpha,\beta) = P_1(\gamma)$, $P_2(\alpha,\beta) = P_2(\gamma)$, $\gamma = \sqrt{\alpha^2+\beta^2}$, therefore

$$T^{(1)} = A_5\left[P_1(\gamma)\sin\omega t + P_2(\gamma)\cos\omega t\right]. \tag{4}$$

If heat sources $A_6 \sin\omega t\sin\alpha x\sin\beta y$ are applied to the surface of the foundation, then

$$T^{(2)} = A_6\left[P_1(\gamma)\cos\omega t - P_2(\gamma)\sin\omega t\right]. \tag{5}$$

For brevity of notation, introduce the designations

$$P_1[\lambda h\gamma^2+\delta] - c\omega P_2 + 1 = \Phi_1, \qquad P_2[\lambda h\gamma^2+\delta] + c\omega P_1 = \Phi_2. \tag{6}$$

Then

$$\begin{gathered} A_5 = \frac{A_1\Phi_1 + A_2\Phi_2}{\Phi_1^2+\Phi_2^2}, \qquad A_6 = \frac{-A_1\Phi_2 + A_2\Phi_1}{\Phi_1^2+\Phi_2^2},\\ A_3 = A_1\frac{\Phi_1 P_1 + \Phi_2 P_2}{\Phi_1^2+\Phi_2^2} + A_2\frac{\Phi_2 P_1 - \Phi_1 P_2}{\Phi_1^2+\Phi_2^2},\\ A_4 = A_1\frac{\Phi_1 P_2 + \Phi_2 P_1}{\Phi_1^2+\Phi_2^2} + A_2\frac{\Phi_2 P_2 - \Phi_1 P_1}{\Phi_1^2+\Phi_2^2}. \end{gathered} \tag{7}$$

It is not difficult to verify that, in essence without loss of generality, one may restrict oneself to the consideration of the case $A_1 \neq 0$, $A_2 = 0$; then the sought function T will have the form

$$T = \frac{A_1 \sin \alpha x \sin \beta y}{\Phi_1^2 + \Phi_2^2} \{[\Phi_1 P_1 + \Phi_2 P_2] \sin \omega t + [\Phi_1 P_2 + \Phi_2 P_1] \cos \omega t\}; \tag{8}$$

if $q_3 = \sum \sum A_{1mn} \sin \alpha_m x \sin \beta_n y$, then

$$T = \sum \sum \frac{A_{1mn} \sin \alpha_m x \sin \beta_n y}{\Phi_{1mn}^2 + \Phi_{2mn}^2} \{[\Phi_1 P_1 + \Phi_2 P_2] \sin \omega t + [\Phi_1 P_2 + \Phi_2 P_1] \cos \omega t\}. \tag{9}$$

If a point heat source $Q \sin \omega t$ acts on the plate, then

$$T = \frac{Q}{\pi^2} \left\{ \sin \omega t \int_0^\infty \int_0^\infty \frac{\cos \alpha x \cos \beta y \, d\alpha \, d\beta}{\widetilde{\Phi}_1(\alpha, \beta)} + \cos \omega t \int_0^\infty \int_0^\infty \frac{\cos \alpha x \cos \beta y \, d\alpha \, d\beta}{\widetilde{\Phi}_2(\alpha, \beta)} \right\}, \tag{10}$$

where

$$\widetilde{\Phi}_1(\alpha, \beta) = \frac{\Phi_1^2 + \Phi_2^2}{\Phi_1 P_1 + \Phi_2 P_2}, \qquad \widetilde{\Phi}_2(\alpha, \beta) = \frac{\Phi_1^2 + \Phi_2^2}{\Phi_1 P_2 + \Phi_2 P_1}.$$

This formula can be written in the form

$$T = \frac{Q}{2\pi} \left[\sin \omega t \int_0^\infty \frac{\gamma J_0(\gamma r) \, d\gamma}{\widetilde{\Phi}_1(\gamma)} + \cos \omega t \int_0^\infty \frac{\gamma J_0(\gamma r) \, d\gamma}{\widetilde{\Phi}_2(\gamma)} \right]. \tag{11}$$

Using the results of (1), one can easily analyze a number of other problems concerning the action of heat sources on a plate and on a rod.

To determine the functions $P_1(\alpha, \beta)$, $P_2(\alpha, \beta)$ in the case where the foundation is a homogeneous half-space, it is necessary to consider the problem of heat waves in a half-space with sources $\sin \alpha x \sin \beta y \sin \omega t$ on the boundary. This problem is equivalent to the problem of bending of a semi-infinite beam with two elastic characteristics, loaded at the end.

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CITED LITERATURE

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[2] B. G. Korenev, DAN, 107, No. 2 (1956).
[3] B. G. Korenev, DAN, 112, No. 1 (1957).