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COMMONALITY OF SLIDING, VIBRATIONAL, AND OPTIMAL MODES OF A CLASS OF SERVOMECHANISMS

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This note illuminates a commonality inherent in the sliding, vibrational, and optimal modes of servomechanisms, consisting in the identical character of their motion in all three modes. The optimal process of motion is, in a certain sense, limiting for the existence of the first two motions of a servomechanism*.

Although sliding modes[1–7] and, especially, vibrational modes[12–14] in the theory of indirect control have been considered for a relatively long time, the study of these modes has until now been carried out independently. At the same time, a servomechanism with a vibro-contour can have a motion analogous to sliding, and a similar motion can also be obtained from a servomechanism without a vibro-contour—by applying positional, velocity, and other feedbacks. It is shown below that, with the aid of these modes, the optimal mode of motion of servomechanisms can to a certain extent be realized ([8–11,16], etc.).

Let us note that by the *vibrational mode* of a servomechanism we here mean its motion in the presence of a vibro-contour embracing the control device (relay). By the *optimal mode* we mean a monotonic transient process (or one with specified oscillations) of a servomechanism, occurring in the least time under conditions actually present in the form of constraints on the coordinates of the servo system and their derivatives, and constraints on the forces or moments applied to the system.

Analysis of the indicated modes and their comparison make it possible to reveal the unity of the dynamic phenomena inherent in all three modes, consisting in the motion of the control device (relay), close to periodic, with a frequency determined by the parameters of the servomechanism, the type of connections, and the magnitude of the disturbance (mismatch). This special type of motion of the servomechanism is a sliding motion and can be defined in phase space as the motion of the servomechanism toward a stationary state (equilibrium position, self-oscillation) after an applied disturbance, in the vibrational mode of the control device (relay). The output coordinate of the servomechanism may change monotonically or with certain oscillations about the instantaneous state corresponding to the balance of the input signals (from the disturbance and the feedbacks). The control device, meanwhile, throughout the entire section of motion in the vibrational mode, alternately switches into the positions "on –off" or "on plus–off minus."

This definition does not restrict the occurrence and course of sliding motion to the condition of zero amplitudes and infinite frequencies of oscillation of the control device, which may theoretically occur with an unambiguous relay characteristic.

In Fig. 1 are presented block diagrams of an automatic control system for the classical problem, and also of a servomechanism with velocity–

* In the present work, use is made of V. V. Petrov' s report at the conference on automatic control in the city of Liblice, Czechoslovakia, in April 1956.

···and rigid feedbacks, and of a servomechanism with two types of vibrocontours.

The first scheme is described, for free oscillations, by the differential equations $T_a\dot{\varphi} = -\mu$; $\delta\eta = \varphi + \frac{\varepsilon}{2}\operatorname{sgn}\dot{\eta}\,(\dot{\eta} \gtrless 0)$; $|\delta\eta - \varphi| < \frac{\varepsilon}{2}$; $(\eta = 0)$ (or, in the absence of dry friction $(\varepsilon = 0)$: $\delta\eta = \varphi)$; $\sigma = \eta - \gamma\mu$; $T_s\dot{\mu} = \Phi(\sigma)$, where $T_a, T_s, \delta, \gamma, \varepsilon$ are constant coefficients; $\varphi, \eta, \sigma, \mu$ are the coordinates of the system; Φ is a relay function, in the general case multivalued.

Fig. 1. Modifications of the structural diagrams of servomechanisms with identical dynamic properties. *I* –automatic control system; *II* –servomechanism with rigid and velocity feedbacks; *III* –servomechanism with a vibrocontour; *IV* –servomechanism with an inertial vibrocontour

The second scheme: $R\dot{\mu} = -\Phi(\mu, \dot{\mu})$, where Φ is likewise a relay function, in general with a nonlinear argument, for realizing an optimal process. In the simplest case of linear connections: $R\dot{\mu} = -\Phi(\gamma\mu + T\dot{\mu})$; R, γ, T are constant coefficients, μ is the output coordinate.

The third scheme: $\gamma\mu + \varkappa\rho = -\sigma$; $T_1\dot{\rho} = \Phi(\sigma)$; $T_2\dot{\mu} = \rho$ for the first type of vibrocontour, and $\gamma\mu + \varkappa\rho = -\sigma$; $T_1\dot{\rho} = \Phi(\sigma)$; $R\dot{\mu} = \Phi(\sigma)$ for the second type of vibrocontour, where $T_1, T_2, \varkappa$ are constant coefficients; ρ, σ, μ are the coordinates of the system. Comparing the equations of motion for the first three schemes $(\varepsilon = 0)$, we observe that their form coincides completely when written with respect to the variables φ and μ. For the scheme with an inertial vibrocontour (Fig. 1, *IV*), the equations of motion correspond to the preceding ones to within a constant of integration $(T_1\rho = B\dot{\mu} + C)$. In the case where $R/T_1 = T_2$, the constant of integration will not introduce any changes into the dynamics of scheme *IV* as compared with scheme *III* (Fig. 1). We shall confine ourselves here to this case.

What has been set forth makes it possible to establish the commonality of the dynamic phenomena occurring in each of the described systems.

If the angle of inclination of the tangent to the trajectory of the representative point moving in phase space at the instant of transition from sheet to sheet is less than or equal to the angle of inclination of the tangent to the trajectory

Fig. 2

Figure 1: Fig. 2

passing through the origin of coordinates, then a sliding regime arises in the system—at first, possibly, with a deviation from the switching line (strip), and then along (inside) it (the necessary and sufficient condition for the occurrence of a sliding regime). The trajectory passing through the origin of coordinates will be called limiting; under one or another set of constraints it corresponds to the optimal regime of motion.

For the examples considered, in the case of an ideal relay characteristic, the necessary and sufficient conditions for the occurrence of a sliding regime are: for the first scheme $|y_0| < 2\delta\gamma/T_s$; for the second $|y_0| < 2T/\gamma R$; for the third $|y_0| < 2\varkappa/\gamma T_1$, and for the fourth

$$|y_0| < \varkappa/\gamma T_1 + \sqrt{\varkappa_1^2/(\gamma T_1)^2 - 2\varkappa C/\gamma R T_1},$$

where y_0 is the value of the ordinate of the representative point at the instant of transition from sheet to sheet*.

* The value of C, as the origin of coordinates is approached, decreases (at the origin of coordinates it is equal to zero), and the condition for scheme IV increasingly approaches the conditions for the first three schemes. For the case $\varepsilon \neq 0$, see (15).

In Fig. 2A a sequence of segments of trajectories is shown, traced by the representative point as it moves in the sliding regime on the phase plane. The region of deviations (in the coordinates x or y) for which sliding motion arises is twice as large as the region of deviations according to the sufficient conditions, which is determined by the phase trajectory passing through the point $y^* \leq T/R\gamma$ (scheme II).

Fig. 2. **A** —a two-sheeted phase surface and trajectories of the sliding regime of a servomechanism: **I** —limiting parabola for the onset of the sliding regime. **B** —a sliding regime approaching the optimal process: **I** —trajectory of the sliding regime; **II** —trajectory of the optimal process

The structure of the multi-sheeted phase surfaces corresponding to systems I—V that are in a sliding regime can be studied completely on the basis of an investigation of the phase trajectories $\Psi(y, x)$ in the switching regions $F = \sigma_i$, where $F(y, x)$ is the argument of the relay function; σ is the relay coordinate.

Then:

1. In the case of ambiguity of the relay function Φ, the system, after some deviation from the strip—without, however, going beyond the limiting trajectory—

Fig. 3

Figure 2: Fig. 3

Fig. 3. Construction of curves that are majorants and minorants for the sliding motion of a servomechanism as functions of time, according to a given "phase portrait" : **I** –majorant and **II** –minorant–exponents of the switching strip; **III** –trajectory of the sliding motion; **IV** –stationary self-oscillatory regime; **V** – limiting cycle

continues to move further in the strip $F = \sigma_i$ with a variable decreasing frequency of the switching device (Fig. 3). The stationary state of the system is a self-oscillatory process with amplitude and frequency determined in (7).

2. In the case of a single-valued characteristic Φ, the system, likewise after some deviation, then moves further along the switching line $F = 0$; in practical calculation this motion requires additional specification.

In the case where Φ is a function of a linear argument, the motion of the system on a segment along the switching line is "linearized" ; its phase trajectory is a straight line determined by the argument F. The order of the equations describing the motion of the system is reduced by at least one.

3. In the case of a nonlinear argument Φ, for which $F(y, x) = \Psi_1(y, x)$, where $\Psi_1(0, 0) = 0$, the sliding mode of motion coincides with the optimal one. The optimal process may therefore be regarded as a limiting (and at the same time particular) case of a sliding mode with switching frequency equal to zero. The transition to the optimal process from sliding is continuous, with a continuous change of the argument F of the switching function Φ to the argument Ψ_1 corresponding to the optimal process. The transition from a sliding mode to the optimal one is characterized by a change in the switching frequency from infinity (for an ideal relay characteristic) to zero.
4. On the basis of the foregoing, the realization of optimal processes is possible, with a certain degree of accuracy, by selecting the parameters of the servomechanism that determine the position of the switching line or strip in the phase plane according to the necessary and sufficient conditions for the existence of a sliding mode, and also by a piecewise-linear approximation of the argument F of the function Φ to the argument Ψ_1. Figure 2b presents an example of a piecewise-linear realization of the relations of an optimal process, and Fig. 3 (time sweep) shows the construction of the majorants and minorants of the transient process of a servomechanism in a sliding mode in the case of a linear argument and a real loop characteristic of the switching device.
5. Thus, servomechanisms with feedbacks in the coordinate and in its time derivatives, and servomechanisms with vibrational loops, make it possible

to tune them to a sliding mode of motion approaching the optimal one. Sliding, vibrational, and optimal modes in the considered systems of indirect control and servomechanism admit a continuous transition in their modes from one to another and are characterized by the commonality of dynamic phenomena caused by motion of the switching device close to periodic, with a frequency decreasing as the optimal process is approached.

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Note: Figure translations are in progress. See original paper for figures.