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Abstract

Full Text

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THEORY OF ELASTICITY

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ELASTIC-PLASTIC DEFORMATION OF A PLANE CAUSED BY THE ACTION OF A CONCENTRATED FORCE

(Presented by Academician S. A. Khristianovich on 5 III 1957)

Below, a solution will be given, in a plastic formulation, of one of the classical problems of the theory of elasticity—on the distribution of stresses and strains in a plane caused by the action of a concentrated force.

Under the condition of plane strain, the solution of the elastic problem has the form ([1])

$$\sigma_r = -\frac{P(3-2\nu)\cos\theta}{4\pi(1-\nu)}\frac{1}{r}, \qquad \sigma_\theta = \frac{P(1-2\nu)\cos\theta}{4\pi(1-\nu)}\frac{1}{r}, \qquad \tau = \frac{P(1-2\nu)\sin\theta}{4\pi(1-\nu)}\frac{1}{r}, \tag{1}$$

where P is the magnitude of the force, referred to unit thickness of the plate; ν is Poisson' s ratio; the arrangement of the polar coordinates is shown in Fig. 1.

The required solution of the elastic-plastic problem will be constructed taking into account the linear hardening law in the plastic region and the incompressibility of the material in the elastic and plastic regions.

The stress and strain components in the elastic and plastic regions must satisfy the system of equilibrium equations

$$\frac{\partial}{\partial r}(r\sigma_r) + \frac{\partial \tau}{\partial \theta} - \sigma_\theta = 0, \qquad \frac{\partial \sigma_\theta}{\partial \theta} + \frac{1}{r}\frac{\partial}{\partial r}(r^2\tau) = 0 \tag{2}$$

and the strain compatibility condition

$$\frac{\partial^2 \varepsilon_r}{\partial \theta^2} - r\frac{\partial \varepsilon_r}{\partial r} + \frac{\partial}{\partial r}\left(r^2\frac{\partial \varepsilon_\theta}{\partial r} - r\frac{\partial \gamma}{\partial \theta}\right) = 0. \tag{3}$$

The relation between the strain and stress components in the plastic region is written by the equations

$$\varepsilon_r = -\varepsilon_\theta = \frac{\psi}{4}(\sigma_r - \sigma_\theta); \tag{4}$$

ψ is an unknown function; for the elastic region $\psi = 1$.

The boundary conditions of the problem reduce to the vanishing of the stress state at infinity. At the boundary of the elastic and plastic regions, the condition of continuity of the stress and strain components must be satisfied and, in addition, $\psi = 1$. The stress intensity σ_i and the shear-strain intensity ε_i are related by the relation

$$\varepsilon_i = \frac{\psi}{2}\sigma_i, \tag{5}$$

which follows from equations (4).

The hardening of the material is approximated by the linear function

$$\sigma_i = 2n\varepsilon_i + \mu, \tag{6}$$

where n and μ are constants connected by the relation

$$n + \mu = 1. \tag{7}$$

The stress components and the force P are referred to the magnitude of the yield limit in pure shear k, and the strain components—to the magnitude k/G.

We shall obtain the required system of stress components in the elastic and plastic regions by putting in (1) $\nu = 0.5$ (the incompressibility condition). We obtain

$$\sigma_r = -\frac{P}{\pi}\frac{\cos\theta}{r}, \qquad \sigma_\theta = \tau = 0. \tag{8}$$

Then the stress intensity will be represented by the expression

$$\sigma_i = \pm\frac{\sigma_r}{2} = \mp\frac{P}{2\pi}\frac{\cos\theta}{r}, \tag{9}$$

where the upper sign refers to the upper half-plane, and the lower sign to the lower half-plane.

Solving the system of equations (5), (6) and taking (9) into account, we find

Fig. 1. Elastic region; plastic region, tension; plastic region, compression; P; M.

Figure 1: Fig. 1. Elastic region; plastic region, tension; plastic region, compression; P; M.

$$\psi = \frac{1}{n}\left(1 - \frac{M}{\mp\frac{P}{2\pi}\frac{\cos\theta}{r}}\right), \tag{10}$$

$$\varepsilon_r = -\varepsilon_\theta = \mp\frac{1}{2n}\left(M \pm \frac{P}{2\pi}\frac{\cos\theta}{r}\right). \tag{11}$$

Fig. 1

The solution obtained (8), (11) exactly satisfies equations (2), (3) and the boundary conditions. The boundary of the plastic region consists of two circles tangent at the point of application of the force. The diameter of the circle of the plastic region is determined from the equation

$$d_s = \mp\frac{r}{\cos\theta} = \frac{P}{2\pi}. \tag{12}$$

In Fig. 1 the plastic region is represented by the shaded circles, the upper circle representing the plastic region of tension, and the lower one that of compression.

The solution obtained is a generalization of the elastoplastic problem for a half-plane with a concentrated force applied on its free boundary [2]. The strains in the elastic region have the form

$$\varepsilon_r = -\varepsilon_\theta = -\frac{P}{4\pi}\frac{\cos\theta}{r}. \tag{13}$$

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CITED LITERATURE

[1] A. Love, *Mathematical Theory of Elasticity*, 1933, p. 220. [2] K. N. Shevchenko, DAN, 61, No. 1 (1948).

Note: Figure translations are in progress. See original paper for figures.